GATE
ELECTRONICS & COMMUNICATION
Vol 8 of 10
To Our Parents
Preface to the Series

For almost a decade, we have been receiving tremendous responses from GATE aspirants for our earlier books: GATE Multiple Choice Questions, GATE Guide, and the GATE Cloud series. Our first book, GATE Multiple Choice Questions (MCQ), was a compilation of objective questions and solutions for all subjects of GATE Electronics & Communication Engineering in one book. The idea behind the book was that Gate aspirants who had just completed or about to finish their last semester to achieve his or her B.E/B.Tech need only to practice answering questions to crack GATE. The solutions in the book were presented in such a manner that a student needs to know fundamental concepts to understand them. We assumed that students have learned enough of the fundamentals by his or her graduation. The book was a great success, but still there were a large ratio of aspirants who needed more preparatory materials beyond just problems and solutions. This large ratio mainly included average students.

Later, we perceived that many aspirants couldn’t develop a good problem solving approach in their B.E/B.Tech. Some of them lacked the fundamentals of a subject and had difficulty understanding simple solutions. Now, we have an idea to enhance our content and present two separate books for each subject: one for theory, which contains brief theory, problem solving methods, fundamental concepts, and points-to-remember. The second book is about problems, including a vast collection of problems with descriptive and step-by-step solutions that can be understood by an average student. This was the origin of GATE Guide (the theory book) and GATE Cloud (the problem bank) series: two books for each subject. GATE Guide and GATE Cloud were published in three subjects only.

Thereafter we received an immense number of emails from our readers looking for a complete study package for all subjects and a book that combines both GATE Guide and GATE Cloud. This encouraged us to present GATE Study Package (a set of 10 books: one for each subject) for GATE Electronic and Communication Engineering. Each book in this package is adequate for the purpose of qualifying GATE for an average student. Each book contains brief theory, fundamental concepts, problem solving methodology, summary of formulae, and a solved question bank. The question bank has three exercises for each chapter: 1) Theoretical MCQs, 2) Numerical MCQs, and 3) Numerical Type Questions (based on the new GATE pattern). Solutions are presented in a descriptive and step-by-step manner, which are easy to understand for all aspirants.

We believe that each book of GATE Study Package helps a student learn fundamental concepts and develop problem solving skills for a subject, which are key essentials to crack GATE. Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge all constructive comments, criticisms, and suggestions from the users of this book. You may write to us at rajkumar. kanodia@gmail.com and ashish.murolia@gmail.com.

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We would like to express our sincere thanks to all the co-authors, editors, and reviewers for their efforts in making this project successful. We would also like to thank Team NODIA for providing professional support for this project through all phases of its development. At last, we express our gratitude to God and our Family for providing moral support and motivation.

We wish you good luck !
R. K. Kanodia
Ashish Murolia
GENERAL ABILITY

Verbal Ability: English grammar, sentence completion, verbal analogies, word groups, instructions, critical reasoning and verbal deduction.

Numerical Ability: Numerical computation, numerical estimation, numerical reasoning and data interpretation.

Engineering Mathematics

Linear Algebra: Matrix Algebra, Systems of linear equations, Eigen values and eigen vectors.

Calculus: Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, Partial Derivatives, Maxima and minima, Multiple integrals, Fourier series. Vector identities, Directional derivatives, Line, Surface and Volume integrals, Stokes, Gauss and Green’s theorems.

Differential equations: First order equation (linear and nonlinear), Higher order linear differential equations with constant coefficients, Method of variation of parameters, Cauchy’s and Euler’s equations, Initial and boundary value problems, Partial Differential Equations and variable separable method.

Complex variables: Analytic functions, Cauchy’s integral theorem and integral formula, Taylor’s and Laurent’ series, Residue theorem, solution integrals.

Probability and Statistics: Sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Discrete and continuous distributions, Poisson, Normal and Binomial distribution, Correlation and regression analysis.


Transform Theory: Fourier transform, Laplace transform, Z-transform.

Electronics and Communication Engineering


**Digital Circuits**: Boolean algebra, minimization of Boolean functions; logic gates; digital IC families (DTL, TTL, ECL, MOS, CMOS). Combinatorial circuits: arithmetic circuits, code converters, multiplexers, decoders, PROMs and PLAs. Sequential circuits: latches and flip-flops, counters and shift-registers. Sample and hold circuits, ADCs, DACs. Semiconductor memories. Microprocessor(8085): architecture, programming, memory and I/O interfacing.


**Control Systems**: Basic control system components; block diagrammatic description, reduction of block diagrams. Open loop and closed loop (feedback) systems and stability analysis of these systems. Signal flow graphs and their use in determining transfer functions of systems; transient and steady state analysis of LTI control systems and frequency response. Tools and techniques for LTI control system analysis: root loci, Routh-Hurwitz criterion, Bode and Nyquist plots. Control system compensators: elements of lead and lag compensation, elements of Proportional-Integral-Derivative (PID) control. State variable representation and solution of state equation of LTI control systems.

**Communications**: Random signals and noise: probability, random variables, probability density function, autocorrelation, power spectral density. Analog communication systems: amplitude and angle modulation and demodulation systems, spectral analysis of these operations, superheterodyne receivers; elements of hardware, realizations of analog communication systems; signal-to-noise ratio (SNR) calculations for amplitude modulation (AM) and frequency modulation (FM) for low noise conditions. Fundamentals of information theory and channel capacity theorem. Digital communication systems: pulse code modulation (PCM), differential pulse code modulation (DPCM), digital modulation schemes: amplitude, phase and frequency shift keying schemes (ASK, PSK, FSK), matched filter receivers, bandwidth consideration and probability of error calculations for these schemes. Basics of TDMA, FDMA and CDMA and GSM.

**Electromagnetics**: Elements of vector calculus: divergence and curl; Gauss’ and Stokes’ theorems, Maxwell’s equations; differential and integral forms. Wave equation, Poynting vector. Plane waves: propagation through various media; reflection and refraction; phase and group velocity; skin depth. Transmission lines: characteristic impedance; impedance transformation; Smith chart; impedance matching; S parameters, pulse excitation. Waveguides: modes in rectangular waveguides; boundary conditions; cut-off frequencies; dispersion relations. Basics of propagation in dielectric waveguide and optical fibers. Basics of Antennas: Dipole antennas; radiation pattern; antenna gain.

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GATE Electronics & Communications Control Systems

Basic control system components; block diagrammatic description, reduction of block diagrams. Open loop and closed loop (feedback) systems and stability analysis of these systems. Signal flow graphs and their use in determining transfer functions of systems; transient and steady state analysis of LTI control systems and frequency response. Tools and techniques for LTI control system analysis: root loci, Routh-Hurwitz criterion, Bode and Nyquist plots. Control system compensators: elements of lead and lag compensation, elements of Proportional-Integral-Derivative (PID) control. State variable representation and solution of state equation of LTI control systems.

IES Electronics & Telecommunication Control Systems

Transient and steady state response of control systems; Effect of feedback on stability and sensitivity; Root locus techniques; Frequency response analysis. Concepts of gain and phase margins: Constant-M and Constant-N Nichol’s Chart; Approximation of transient response from Constant-N Nichol’s Chart; Approximation of transient response from closed loop frequency response; Design of Control Systems, Compensators; Industrial controllers.

IAS Electrical Engineering Control Systems

Elements of control systems; block-diagram representations; open-loop & closed-loop systems; principles and applications of feed-back. LTI systems : time domain and transform domain analysis. Stability : Routh Hurwitz criterion, root-loci, Nyquist’s criterion. Bode-plots, Design of lead-lag compensators; Proportional, PI, PID controllers.
CHAPTER 1  TRANSFER FUNCTIONS

1.1 INTRODUCTION 1

1.2 CONTROL SYSTEM 1
   1.2.1 Classification of Control System 1
   1.2.2 Mathematical Modelling of Control System 3

1.3 TRANSFER FUNCTION 3

1.4 FEEDBACK SYSTEM 4
   1.4.1 Basic Formulation 5
   1.4.2 Transfer Function for Multivariable System 5
   1.4.3 Effects of Feedback on System Characteristics 6

1.5 BLOCK DIAGRAMS 7
   1.5.1 Block Diagram Reduction 7

1.6 SIGNAL FLOW GRAPH 10
   1.6.1 Representation of Signal Flow Graph 10
   1.6.2 Basic Terminologies of SFG 10
   1.6.3 Gain Formula for SFG (Mason’s Rule) 10

EXERCISE 1.1 12
EXERCISE 1.2 33
EXERCISE 1.3 37
SOLUTIONS 1.1 45
SOLUTIONS 1.2 86
SOLUTIONS 1.3 93

CHAPTER 2  STABILITY

2.1 INTRODUCTION 99

2.2 LTI SYSTEM RESPONSES 99

2.3 STABILITY 99
CHAPTER 4  ROOT LOCUS TECHNIQUE

4.1  INTRODUCTION  287
4.2  ROOT LOCUS  287
   4.2.1  The Root-Locus Concept  287
   4.2.2  Properties of Root Locus  288
4.3  RULES FOR SKETCHING ROOT LOCUS  289
4.4  EFFECT OF ADDITION OF POLES AND ZEROS TO G(S)H(S)  291
4.5  ROOT SENSITIVITY  291

EXERCISE 4.1  292
EXERCISE 4.2  310
EXERCISE 4.3  314
SOLUTIONS 4.1  320
SOLUTIONS 4.2  352
SOLUTIONS 4.3  364

CHAPTER 5  FREQUENCY DOMAIN ANALYSIS

5.1  INTRODUCTION  369
5.2  FREQUENCY RESPONSE  369
   5.2.1  Correlation Between Time and Frequency Response  370
   5.2.2  Frequency Domain Specifications  370
   5.2.3  Effect of Adding a Pole or a Zero to Forward Path Transfer Function  371
5.3  POLAR PLOT  372
5.4 NYQUIST CRITERION 372
5.4.1 Principle of Argument 372
5.4.2 Nyquist Stability Criterion 373
5.4.3 Effect of Addition of Poles and Zeros to \( G(s)H(s) \) on Nyquist Plot 375

5.5 BODE PLOTS 375
5.5.1 Initial Part of Bode Plot 375
5.5.2 Slope Contribution of Poles and Zeros 376
5.5.3 Determination of Steady State Error Characteristics 376

5.6 ALL-PASS AND MINIMUM PHASE SYSTEM 377
5.6.1 Pole-Zero Pattern 377
5.6.2 Phase Angle Characteristic 378

5.7 SYSTEM WITH TIME DELAY (TRANSPORTATION LAG) 378

5.8 GAIN MARGIN AND PHASE MARGIN 379
5.8.1 Determination of Gain Margin and Phase Margin using Nyquist Plot 379
5.8.2 Determination of Gain Margin and Phase Margin using Bode Plot 380
5.8.3 Stability of a System 381

5.9 CONSTANT \( M \)-CIRCLES AND CONSTANT \( N \)-CIRCLES 381
5.9.1 \( M \)-Circles 381
5.9.2 \( N \)-Circles 382

5.10 NICHOLS CHARTS 383
EXERCISE 5.1 384
EXERCISE 5.2 406
EXERCISE 5.3 411
SOLUTIONS 5.1 426
SOLUTIONS 5.2 465
SOLUTIONS 5.3 481

CHAPTER 6  DESIGN OF CONTROL SYSTEMS

6.1 INTRODUCTION 491

6.2 SYSTEM CONFIGURATIONS 491

6.3 CONTROLLERS 492
6.3.1 Proportional Controller 492
6.3.2 Proportional-Derivative (PD) Controller 493
CHAPTER 7  STATE VARIABLE ANALYSIS

7.1  INTRODUCTION  559

7.2  STATE VARIABLE SYSTEM  559
7.2.1  State Differential Equations  560
7.2.2  Block Diagram of State Space  560
7.2.3  Comparison between Transfer Function Approach and State Variable Approach  561

7.3  STATE-SPACE REPRESENTATION  561
7.3.1  State-Space Representation using Physical Variables  561
7.3.2  State-Space Representation Using Phase Variable  562

7.4  SOLUTION OF STATE EQUATION  563
7.4.1  Solution of Non-homogeneous State Equation  563
7.4.2  State Transition Matrix by Laplace Transform  564

7.5  TRANSFER FUNCTION FROM THE STATE MODEL  565
7.5.1  Characteristic Equation  565
7.5.2  Eigen Values  565
7.5.3  Eigen Vectors  566
7.5.4  Determination of Stability Using Eigen Values  566

7.6  SIMILARITY TRANSFORMATION  566
7.6.1  Diagonalizing a System Matrix  566
## 7.7 CONTROLLABILITY AND OBSERVABILITY 566

7.7.1 Controllability 567
7.7.2 Output Controllability 567
7.7.3 Observability 567

## 7.8 STATE FEEDBACK CONTROL SYSTEM 568

## 7.9 STEADY STATE ERROR IN STATE SPACE 568

7.9.1 Analysis Using Final Value Theorem 568
7.9.2 Analysis Using Input Substitution 569

### EXERCISE 7.1 571
### EXERCISE 7.2 599
### EXERCISE 7.3 601
### SOLUTIONS 7.1 606
### SOLUTIONS 7.2 664
### SOLUTIONS 7.3 669

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1.1 INTRODUCTION

The performance of a feedback system can be described in terms of the location of the roots of the characteristic equation, graphically, in the $s$-plane. This qualitative nature of the solution will be examined in this chapter using the root-locus analysis. Following topics are included in the chapter:

- Basic concept of the root locus method
- Useful rules for constructing the root loci
- Effect of adding poles and zeros to $G(s)H(s)$
- Root sensitivity

1.2 ROOT LOCUS

A graph showing how the roots of the characteristic equation move around the $s$-plane as a single parameter varies is known as a root locus plot.

1.2.1 The Root-Locus Concept

The roots of the characteristic equation of a system provide a valuable concern concerning the response of the system. To understand the root locus concept, consider the characteristics equation

$$q(s) = 1 + G(s)H(s) = 0$$

Now, we rearrange the equation so that the parameter of interest, $K$, appears as the multiplying factor in the form,

$$1 + KP(s) = 0$$

For determining the locus of roots as $K$ varies from 0 to $\infty$, consider the polynomial in the form of poles and zeros as

$$1 + \frac{K \prod_{i} (s + Z_{i})}{\prod_{j} (s + P_{j})} = 0$$

or

$$\prod_{j} (s + P_{j}) + K \prod_{i} (s + Z_{i}) = 0$$

when $K = 0$, the roots of the characteristic equation are the poles of $P(s)$.

i.e.

$$\prod_{j} (s + P_{j}) = 0$$

when $K = \infty$, the roots of the characteristic equation are the zeros of $P(s)$.

i.e.

$$\prod_{i} (s + Z_{i}) = 0$$

Hence, we noted that the locus of the roots of the characteristic equation $1 + KP(s) = 0$ begins at the poles of $P(s)$ and ends at the zeros of $P(s)$ as $K$ increases from zero to infinity.
1.2.2 Properties of Root Locus

To examine the properties of root locus, we consider the characteristic equation as

\[ 1 + G(s)H(s) = 0 \]

or

\[ 1 + KG_1(s)H_1(s) = 0 \]

where \( G_1(s)H_1(s) \) does not contain the variable parameter \( K \). So, we get

\[ G_1(s)H_1(s) = -\frac{1}{K} \]

From above equation, we conclude the following result:

1. For any value of \( s \) on the root locus, we have the magnitude

\[ |G_1(s)H_1(s)| = \frac{1}{|K|}; \quad -\infty < K < \infty \]

2. For any value of \( s \) on the root locus, we have

\[ \arg(G_1(s)H_1(s)) = (2k + 1)\pi; \quad \text{where} \ k = 0, \pm 1, \pm 2, \ldots \]

\[ \text{odd multiple of } 180^\circ \]

for \( 0 \leq K < \infty \)

\[ \arg(G_1(s)H_1(s)) = 2k\pi; \quad \text{where} \ k = 0, \pm 1, \pm 2, \ldots \]

\[ \text{even multiple of } 180^\circ \]

for \( -\infty < K \leq 0 \)

3. Once the root locus are constructed, the values of \( K \) along the loci can be determined by

\[ K = \frac{\prod_{j=1}^{n} |(s + P_j)|}{\prod_{i=1}^{m} |(s + Z_i)|} \]

The value of \( K \) at any point \( s_1 \) on the root locus is obtained from above equation by substituting value of \( s_1 \). Graphically, we write

\[ K = \text{Product of vector lengths drawn from the poles of } G(s)H(s) \text{ to } s_1 \]

\[ \text{Product of vector lengths drawn from the zeros of } G(s)H(s) \text{ to } s_1 \]

### POINTS TO REMEMBER

1. Root loci are trajectories of roots of characteristic equation when a system parameter varies.
2. In general, this method can be applied to study the behaviour of roots of any algebraic equation with one or more variable parameters.
3. Root loci of multiple variable parameters can be treated by varying one parameter at a time. The resultant loci are called the root contours.
4. Root-Loci refers to the entire root loci for \(-\infty < K < \infty\).
5. In general, the values of \( K \) are positive \( (0 < K < \infty) \). Under unusual conditions when a system has positive feedback or the loop gain is negative, then we have \( K \) as negative.
1.3 RULES FOR SKETCHING ROOT LOCUS

Some important rules are given in the following texts that are useful for sketching the root loci.

Rule 1: Symmetry of Root Locus

Root locus are symmetrical with respect to the real axis of the s-plane. In general, the root locus are symmetrical with respect to the axes of symmetry of the pole-zero configuration of \( G(s)H(s) \).

Rule 2: Poles and Zeros on the Root Locus

To locate the poles and zeros on root locus, we note the following points.
1. The \( K = 0 \) points on the root loci are at the poles of \( G(s)H(s) \).
2. The \( K = \pm \infty \) points on the root loci are at zeros of \( G(s)H(s) \).

Rule 3: Number of Branches of Root Locus

The number of branches of the root locus equals to the order of the characteristic polynomial.

Rule 4: Root Loci on the Real axis

While sketching the root locus on real axis, we must note following points:
1. The entire real axis of the s-plane is occupied by the root locus for all values of \( K \).
2. Root locus for \( K \geq 0 \) are found in the section only if the total number of poles and zeros of \( G(s)H(s) \) to the right of the section is odd. The remaining sections of the real axis are occupied by the root locus for \( K \leq 0 \).
3. Complex poles and zeros of \( G(s)H(s) \) do not affect the type of root locus found on the real axis.

Rule 5: Angle of Asymptotes of the Root Locus

When \( n \) is the number of finite poles and \( m \) is the number of finite zeros of \( G(s)H(s) \), respectively. Then \( |n - m| \) branches of the root locus approaches the infinity along straight line asymptotes whose angles are given by

\[
\theta_a = \pm \left( \frac{2q + 1}{n - m} \right) \pi; \quad \text{for } K \geq 0
\]

and

\[
\theta_a = \pm \left( \frac{2q}{n - m} \right) \pi; \quad \text{for } K \leq 0
\]

where \( q = 0, 1, 2, \ldots \ldots (n - m - 1) \)

Rule 6: Determination of Centroid

The asymptotes cross the real axis at a point known as centroid, which is given by

\[
\sigma_A = \sum \text{Real parts of poles of } G(s)H(s) - \sum \text{Real parts of zeros of } G(s)H(s)
\]

Rule 7: Angle of Departure

The angle of departure from an open loop pole is given by (for \( K \geq 0 \))

\[
\phi_0 = \pm \left( (2q + 1)\pi + \phi \right); \quad q = 0, 1, 2, \ldots \ldots
\]

where, \( \phi \) is the net angle contribution at this pole, of all other open loop poles and zeros. For example, consider the plot shown in figure below.
Figure 4.1: Illustration of Angle of Departure

From the figure, we obtain

\[ \phi = \theta_3 + \theta_5 - (\theta_1 + \theta_2 + \theta_4) \]

and

\[ \phi_D = \pm [(2q + 1)\pi + \phi]; \quad q = 0, 1, 2, \ldots \]

Rule 8: Angle of Arrival

The angle of arrival at an open loop zero is given by (for \( K \geq 0 \))

\[ \phi_a = \pm [(2q + 1)\pi - \phi]; \quad q = 0, 1, 2, \ldots \]

where \( \phi \) = net angle contribution at this zero, of all other open loop poles and zeros. For example, consider the plot shown in figure below.

Figure 4.2: Illustration of Angle of Arrival

From the figure, we obtain

\[ \phi = \theta_2 - (\theta_1 + \theta_3) \]

and

\[ \phi_a = \pm [(2q + 1)\pi - \phi]; \quad q = 0, 1, 2, \ldots \]

**NOTE:**

For \( K \leq 0 \), departure and arrival angles are given by

\[ \phi_D = \mp [(2q + 1)\pi + \phi] \]

and

\[ \phi_a = \mp [(2q + 1)\pi - \phi] \]

Rule 9: Break-away and Break-in Points

To determine the break-away and break-in points on the root locus, we consider the following points:

1. A root locus plot may have more than one breakaway points.
2. Break away points may be complex conjugates in the \( s \)-plane.
3. At the break away or break-in point, the branches of the root locus form an angle of \( \frac{180n}{\pi} \) with the real axis, where \( n \) is the number of closed loop poles arriving at or departing from the single breakaway or break-in point on the real axis.
4. The breakaway and break-in points of the root locus are the solution of
\[
\frac{dK}{ds} = 0
\]

i.e. breakaway and break in points are determined by finding maximum and minimum points of the gain \( K \) as a function of \( s \) with \( s \) restricted to real values.

**Rule 10: Intersects of Root Locus on Imaginary Axis**

Routh-Hurwitz criterion may be used to find the intersects of the root locus on the imaginary axis.

### 1.4 EFFECT OF ADDITION OF POLES AND ZEROS TO \( G(s)H(s) \)

In this section, the effect of adding poles and zeros to \( G(s)H(s) \) are described.

**Addition of Poles to \( G(s)H(s) \)**

Due to addition of poles to \( G(s)H(s) \), the root locus is affected in following manner:
1. Adding a pole to \( G(s)H(s) \) has the effect of pushing the root loci towards the right half.
2. The complex path of the root loci bends to the right.
3. Angle of asymptotes reduces and centroid is shifted to the left.
4. The system stability will be reduced.

**Addition of Zeros to \( G(s)H(s) \)**

Due to addition of zeros to \( G(s)H(s) \), the root locus is affected in following manner:
1. Adding left half plane zero to the function \( G(s)H(s) \), generally has the effect of moving or pushing the root loci towards the left half.
2. The complex path of the root loci bends to the left.
3. Centroid shifted to the right.
4. The relative stability of the system is improved.

### 1.5 ROOT SENSITIVITY

The sensitivity of the roots of the characteristic equation when \( K \) varies is termed as the root sensitivity. Mathematically, the root sensitivity is defined as the ratio of the fractional change in a closed-loop pole to the fractional change in a system parameter, such as gain and is given by

\[
S_K^s = \frac{K}{s} \frac{ds}{dK}
\]

where \( s \) is the current pole location, and \( K \) is the current system gain. Converting the partials to finite increments, the actual change in closed-loop poles can be approximated as

\[
\Delta s = s(S_K^s) \frac{\Delta K}{K}
\]

where, \( \Delta s \) is the change in pole location and \( \Delta K/K \) is the fractional change in the gain, \( K \).

**NOTE:**
The root sensitivity at the breakaway points is infinite, because break away points are given by \( \frac{dK}{ds} = 0 \).
**EXERCISE 1.1**

**MCQ 1.1.1** Form the given sketch the root locus can be

(A) ![Diagram A](image) \hspace{1cm} (B) ![Diagram B](image)

(C) ![Diagram C](image) \hspace{1cm} (D) ![Diagram D](image)

**MCQ 1.1.2** Consider the sketch shown below.

(1) ![Diagram 1](image) \hspace{1cm} (2) ![Diagram 2](image)

(3) ![Diagram 3](image) \hspace{1cm} (4) ![Diagram 4](image)

The root locus can be

(A) (1) and (3) \hspace{1cm} (B) (2) and (3)

(C) (2) and (4) \hspace{1cm} (D) (1) and (4)

**MCQ 1.1.3** The valid root locus diagram is

(A) ![Diagram A](image) \hspace{1cm} (B) ![Diagram B](image)
MCQ 1.1.4

An open-loop pole-zero plot is shown below.

The general shape of the root locus is

(A)  (B)  (C)  (D)

MCQ 1.1.5

An open-loop pole-zero plot is shown below.

The general shape of the root locus is

(A)  (B)  (C)  (D)
MCQ 1.1.6

An open-loop pole-zero plot is shown below.

The general shape of the root locus is

(A) ![Diagram](image1)

(B) ![Diagram](image2)

(C) ![Diagram](image3)

(D) ![Diagram](image4)

MCQ 1.1.7

An open-loop pole-zero plot is shown below.

The general shape of the root locus is

(A) ![Diagram](image5)

(B) ![Diagram](image6)

(C) ![Diagram](image7)

(D) ![Diagram](image8)
MCQ 1.1.8

The forward-path open-loop transfer function of a ufb system is
\[ G(s) = \frac{K(s+2)(s+6)}{s^2 + 8s + 25} \]

The root locus for this system is

(A) \hspace{1cm} (B)

(C) \hspace{1cm} (D)

MCQ 1.1.9

The forward-path open-loop transfer function of a ufb system is
\[ G(s) = \frac{K(s^2 + 4)}{(s^2 + 1)} \]

For this system, root locus is

(A) \hspace{1cm} (B)

(C) \hspace{1cm} (D)

MCQ 1.1.10

The forward-path open-loop transfer function of a ufb system is
\[ G(s) = \frac{K(s^2 + 1)}{s^2} \]

The root locus of this system is
Root Locus Technique

Common Data For Q. 11 and 12
An open-loop pole-zero plot is shown below.

Common Data

MCQ 1.1.11 The transfer function of this system is
(A) \( \frac{K(s^2 + 2s + 2)}{(s + 3)(s + 2)} \)
(B) \( \frac{K(s + 3)(s + 2)}{s^2 + 2s + 2} \)
(C) \( \frac{K(s^2 - 2s + 2)}{(s + 3)(s + 2)} \)
(D) \( \frac{K(s + 3)(s + 2)}{s^2 - 2s + 2} \)

MCQ 1.1.12 The break point is
(A) break away at \( \sigma = -1.29 \)
(B) break in at \( \sigma = -2.43 \)
(C) break away at \( \sigma = -2.43 \)
(D) break in at \( \sigma = -1.29 \)

MCQ 1.1.13 The forward-path transfer function of a \( u/f \) system is
\[
G(s) = \frac{K(s + 1)(s + 2)}{(s + 5)(s + 6)}
\]
So, the break points are
Break-in \( \sigma \)
(A) \(-1.563\) \(-5.437\)
(B) \(-5.437\) \(-1.563\)
(C) \(-1.216\) \(-5.743\)
(D) \(-5.473\) \(-1.216\)
MCQ 1.1.14  Consider the feedback system shown below.

For this system, the root locus is

(A) ![Root Locus Option A](image1)
(B) ![Root Locus Option B](image2)
(C) ![Root Locus Option C](image3)
(D) ![Root Locus Option D](image4)

MCQ 1.1.15  For a \( ufb \) system, forward-path transfer function is

\[ G(s) = \frac{K(s+6)}{(s+3)(s+5)} \]

The breakaway point and break-in points are located respectively at

(A) \( \infty, 4.27 \)  
(B) \( 7.73, 4.27 \)  
(C) \( 4.27, \infty \)  
(D) \( 4.27, 7.73 \)

MCQ 1.1.16  The open loop transfer function of a system is given by

\[ G(s)H(s) = \frac{K}{s(s+1)(s+2)} \]

The root locus plot of above system is

(A) ![Root Locus Option A](image5)
(B) ![Root Locus Option B](image6)
(C) ![Root Locus Option C](image7)
(D) ![Root Locus Option D](image8)

MCQ 1.1.17  A \( ufb \) system has forward-path transfer function,

\[ G(s) = \frac{K}{s} \]
The root locus plot is

(A) \hspace{1cm} (B)

\( j\omega \) \hspace{1cm} \( j\omega \) \hspace{1cm} \( \sigma \) \hspace{1cm} \( \sigma \)

(C) \hspace{1cm} (D)

\( j\omega \) \hspace{1cm} \( j\omega \) \hspace{1cm} \( \sigma \) \hspace{1cm} \( \sigma \)

**MCQ 1.1.18**

For the \textit{ufb} system shown below, consider two points

\( s_1 = -2 + j3 \) and \( s_2 = -2 + j\frac{1}{\sqrt{2}} \)

Which of the above points lie on root locus?

(A) Both \( s_1 \) and \( s_2 \) \hspace{1cm} (B) \( s_1 \) but not \( s_2 \)

(C) \( s_2 \) but not \( s_1 \) \hspace{1cm} (D) neither \( s_1 \) nor \( s_2 \)

**MCQ 1.1.19**

A \textit{ufb} system has open-loop transfer function,

\[ G(s) = \frac{K(s + \alpha)}{s^2(s + \beta)} , \beta > \alpha > 0 \]

The valid root-loci for this system is

(A) \hspace{1cm} (B)

\( j\omega \) \hspace{1cm} \( j\omega \) \hspace{1cm} \( \sigma \) \hspace{1cm} \( \sigma \)

(C) \hspace{1cm} (D)

\( j\omega \) \hspace{1cm} \( j\omega \) \hspace{1cm} \( \sigma \) \hspace{1cm} \( \sigma \)
The characteristic equation of a feedback control system is given by
\[(s^2 + 4s + 4)(s^2 + 11s + 30) + Ks^2 + 4K = 0\]
where \(K > 0\). In the root locus of this system, the asymptotes meet in \(s\)-plane at
(A) \((-9.5,0)\)
(B) \((-5.5,0)\)
(C) \((-7.5,0)\)
(D) None of the above

The root locus of the system having the loop transfer function,
\[G(s)H(s) = \frac{K}{s(s + 4)(s^2 + 4s + 5)}\]
has
(A) 3 break-away points
(B) 3 break-in points
(C) 2 break-in and 1 break-away point
(D) 2 break-away and 1 break-in point

Consider the \(ufb\) system shown below.

The root-loci, as \(\alpha\) is varied, will be

Common Data For Q. 23 and 24

The forward-path transfer function of a \(ufb\) system is
\[G(s) = \frac{K(s + \alpha)(s + 3)}{s(s^2 - 1)}\]
MCQ 1.1.23 The root-loci for $K > 0$ with $\alpha = 5$ is

(A) \hspace{2cm} (B)

(C) \hspace{2cm} (D)

MCQ 1.1.24 The root-loci for $\alpha > 0$ with $K = 10$ is

(A) \hspace{2cm} (B)

(C) \hspace{2cm} (D)

MCQ 1.1.25 For the system $G(s) H(s) = \frac{K(s + 6)}{(s + 2)(s + 4)}$, consider the following characteristic of the root locus:
1. It has one asymptote.
2. It has intersection with $j\omega$-axis.
3. It has two real axis intersections.
4. It has two zeros at infinity.
The root locus have characteristics
(A) 1 and 2 \hspace{2cm} (B) 1 and 3
(C) 3 and 4 \hspace{2cm} (D) 2 and 4

MCQ 1.1.26 The forward path transfer function of a ufb system is

$$G(s) = \frac{K(s + 3)}{s(s + 1)(s + 2)(s + 4)}$$
The angles of asymptotes are

(A) $0, \frac{\pi}{2}, \pi$

(B) $0, \frac{2\pi}{3}, \frac{4\pi}{3}$

(C) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

(D) None of the above

MCQ 1.1.27

Match List-I with List-II in respect of the open loop transfer function

$$G(s)H(s) = \frac{K(s + 10)(s^2 + 20s + 500)}{s(s + 20)(s + 50)(s^2 + 4s + 5)}$$

and choose the correct option.

<table>
<thead>
<tr>
<th>List I (Types of Loci)</th>
<th>List II (Numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. Separate Loci</td>
<td>1. One</td>
</tr>
<tr>
<td>Q. Loci on the real axis</td>
<td>2. Two</td>
</tr>
<tr>
<td>R. Asymptotes</td>
<td>3. Three</td>
</tr>
<tr>
<td>S. Break away points</td>
<td>4. Five</td>
</tr>
</tbody>
</table>

(A) 4 3 1 1

(B) 4 3 2 1

(C) 3 4 1 1

(D) 3 4 1 2

MCQ 1.1.28

The root-locus of a $ufb$ system is shown below.

The open loop transfer function is

(A) $\frac{K}{s(s+1)(s+3)}$

(B) $\frac{K(s+3)}{s(s+1)}$

(C) $\frac{K(s+1)}{s(s+3)}$

(D) $\frac{Ks}{(s+1)(s+3)}$

MCQ 1.1.29

The characteristic equation of a linear control system is $s^2 + 5Ks + 9 = 0$. The root loci of the system is

(A)  

(B)  

(C)  

(D)
An unity feedback system is given as \( G(s) = \frac{K(1 - s)}{s(s + 3)} \). Which of the following is the correct root locus diagram?

(A) \[ \begin{array}{c}
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\end{array} \]

(B) \[ \begin{array}{c}
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\end{array} \]

(C) \[ \begin{array}{c}
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\end{array} \]

(D) \[ \begin{array}{c}
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\end{array} \]

The open loop transfer function \( G(s) \) of a \( ufb \) system is given as

\[
G(s) = \frac{K(s + \frac{1}{2})}{s(s + 2)}
\]

From the root locus, it can be inferred that when \( K \) tends to positive infinity,

(A) three roots with nearly equal real parts exist on the left half of the \( s \)-plane

(B) one real root is found on the right half of the \( s \)-plane

(C) the root loci cross the \( j\omega \) axis for a finite value of \( K; K \neq 0 \)

(D) three real roots are found on the right half of the \( s \)-plane

The characteristic equation of a closed-loop system is

\[
s(s + 1)(s + 3) + K(s + 2) = 0, \quad K > 0
\]

Which of the following statements is true?

(A) Its roots are always real

(B) It cannot have a breakaway point in the range \(-1 < \Re[s] < 0\)

(C) Two of its roots tend to infinity along the asymptotes \( \Re[s] = -1 \)

(D) It may have complex roots in the right half plane.

A closed-loop system has the characteristic function,

\[
(s^2 - 4)(s + 1) + K(s - 1) = 0
\]

Its root locus plot against \( K \) is

(A) \[ \begin{array}{c}
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\end{array} \]

(B) \[ \begin{array}{c}
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\end{array} \]
MCQ 1.1.34  Figure shows the root locus plot (location of poles not given) of a third order system whose open loop transfer function is

(A) $\frac{K}{s^3}$  
(B) $\frac{K}{s^2(s+1)}$  
(C) $\frac{K}{s(s^2+1)}$  
(D) $\frac{K}{s(s^2-1)}$

MCQ 1.1.35  A unity feedback system has an open loop transfer function, $G(s) = \frac{K}{s^2}$. The root locus plot is

(A)  
(B)  
(C)  
(D)  

Common Data For Q. 36 to 38
The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{2(s+\alpha)}{s(s+2)(s+10)}; \alpha > 0$$
Angles of asymptotes are
(A) $60^\circ, 120^\circ, 300^\circ$
(B) $60^\circ, 180^\circ, 300^\circ$
(C) $90^\circ, 270^\circ, 360^\circ$
(D) $90^\circ, 180^\circ, 270^\circ$

Intercepts of asymptotes at the real axis is
(A) $-6$  (B) $\frac{-10}{3}$
(C) $-4$  (D) $-8$

Break away points are
(A) $-1.056, -3.471$  (B) $-2.112, -6.943$
(C) $-1.056, -6.943$  (D) $1.056, -6.943$

For the characteristic equation $s^3 + 2s^2 + Ks + K = 0$, the root locus of the system as $K$ varies from zero to infinity is

The open loop transfer function of a $ufb$ system and root locus plot for the system is shown below.

$$G(s)H(s) = \frac{K}{s(s+2)(s+4)}$$
The range of $K$ for which the system has damped oscillatory response is
(A) $0 < K < 48$  
(B) $K > 3.08$
(C) $3.08 < K < 48$  
(D) $K > 48$

**MCQ 1.1.41**

The root locus plot for a control system is shown below.

Consider the following statements regarding the system.
1. System is stable for all positive value of $K$.
2. System has real and repeated poles for $0.573 < K < 7.464$.
3. System has damped oscillatory response for all values of $K$ greater than $0.573$.
4. System is overdamped for $0 < K < 0.573$ and $K > 7.464$.
Which of the following is correct ?
(A) 1, 2 and 3  
(B) 1 and 4
(C) 2 and 3  
(D) all

**MCQ 1.1.42**

The open loop transfer function of a control system is

$$G(s)H(s) = \frac{Ke^{-s}}{s(s + 2)}$$

For low frequencies, consider the following statements regarding the system.
1. $s = 2.73$ is break-away point.
2. $s = -0.73$ is break-away point.
3. $s = -0.73$ is break-in point.
4. $s = 2.73$ is break-in point.
Which of the following is correct ?
(A) 1 and 2  
(B) 3 and 4
(C) 1 and 3  
(D) 2 and 4
The open loop transfer function of a \( \text{ufb} \) system is
\[ G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+20)} \]
The root locus of the system has
(A) 3 real break point.
(B) 1 real and 2 complex break point.
(C) only one real break point.
(D) No one break point.

The open loop transfer function of a control system is given below.
\[ G(s)H(s) = \frac{Ks}{(s^2-s+4.25)} \]
The system gain \( K \) as a function of \( s \) along real axis is shown below.

In the root locus plot, point \( s \) corresponding to gain plot, is
(A) \( s = -2.06 \); Break in point
(B) \( s = -1.25 \); Break in point
(C) \( s = -2.06 \); Break away point
(D) \( s = -4.25 \); Break away point

Common Data For Q. 45 and 46

The open loop transfer function of a system is
\[ G(s)H(s) = \frac{K(s-1)}{(s+1)(s+2)} \]

What is the gain \( K \) for which the closed loop system has a pole at \( s = 0 \) ?
(A) \( K = 0 \)
(B) \( K = 2 \)
(C) \( K = 10 \)
(D) \( K = \infty \)

Based on the above result, other pole of the system is
(A) \( s = -3 + j0 \)
(B) \( s = -5 + j0 \)
(C) \( s = -13 + j0 \)
(D) None

Common Data For Q. 47 to 49

The open loop transfer function of a system is given below.
\[ G(s)H(s) = \frac{K(s+2)}{s(s+1)(s+30)} \]

What is the value of gain \( K \) for which the closed loop system has two poles with real part \(-2\) ?
(A) \( K = 0 \)
(B) \( K = 30 \)
(C) \( K = 84.24 \)
(D) \( K = \infty \)
MCQ 1.1.48
Based on the above result, third pole of the closed loop system is
(A) \( s = -30 \)  
(B) \( s = -31 \)  
(C) \( s = -27 \)  
(D) None

MCQ 1.1.49
Complex poles of the system are
(A) \( s = -2 \pm 2.245 \)  
(B) \( s = -2 \pm 3.467 \)  
(C) \( s = -2 \pm 1.497 \)  
(D) None

MCQ 1.1.50
Consider the system with delay time \( \tau_d \) shown below.

\[ R(s) \xrightarrow{+} \frac{K}{s} e^{-\tau_d s} \xrightarrow{-} Y(s) \]

Suppose delay time \( \tau_d = 1 \text{ sec} \). In root locus plot of the system, the breakaway and break-in points are respectively
(A) 0, 4.83  
(B) 4.83, 0  
(C) -0.83, 4.83  
(D) 4.83, -0.83

MCQ 1.1.51
Variation of system gain \( K \) along the real axis of \( s \)-plane for the root locus of a system is shown below.

\[ K(s) \]

\[ -3 \quad -2 \quad -\sigma_1 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \text{Re}(s) \]

Which of the following options is correct for the root locus plot of the system?
(A) \((-\sigma_1)\) is break in and \(\sigma_2\) is break away point.
(B) \((-\sigma_1)\) is break away and \(\sigma_2\) is break in point.
(C) Both \((-\sigma_1)\) and \(\sigma_2\) are break away points.
(D) Both \((-\sigma_1)\) and \(\sigma_2\) are break in points.

**Common Data For Q. 52 to 54**
Consider the system given below.

\[ R(s) \xrightarrow{+} \frac{K}{s} \frac{1}{s(s+10)} \xrightarrow{-} C(s) \]
MCQ 1.1.52

Poles of the system for \( K = 5 \) are

(A) \( s = -9.47 \) \hspace{1cm} (B) \( s = -0.53 \) \hspace{1cm} (C) Both A and B \hspace{1cm} (D) None

MCQ 1.1.53

What will be the change in pole \( s = -9.47 \) location for a 10% change in \( K \) ?

(A) Pole moves to left by 0.056 \hspace{1cm} (B) Pole moves to right by 0.056 \hspace{1cm} (C) Pole moves to left by 0.029 \hspace{1cm} (D) Pole moves to right by 0.029

MCQ 1.1.54

A \( \text{ufb} \) system has an open loop transfer function,

\[
G(s)H(s) = \frac{K(s+1)}{s(s-1)}
\]

Root locus for the system is a circle. Centre and radius of the circle are respectively

(A) \( (0,0), 2 \) \hspace{1cm} (B) \( (0,0), \sqrt{2} \) \hspace{1cm} (C) \( (-1,0), 2 \) \hspace{1cm} (D) \( (-1,0), \sqrt{2} \)

MCQ 1.1.55

The open loop transfer function of a system is

\[
G(s)H(s) = \frac{K(s+3)}{s(s+2)}
\]

The root locus of the system is a circle. The equation of circle is

(A) \( (\sigma + 4)^2 + \omega^2 = 4 \) \hspace{1cm} (B) \( (\sigma - 3)^2 + \omega^2 = 3 \) \hspace{1cm} (C) \( (\sigma + 3)^2 + \omega^2 = (\sqrt{3})^2 \) \hspace{1cm} (D) \( (\sigma - 4)^2 + \omega^2 = (2)^2 \)

MCQ 1.1.56

Consider the open loop transfer function of a system given below.

\[
G(s)H(s) = \frac{K}{(s^2 + 2s + 2)(s^2 + 6s + 10)}
\]

The break-away point in root locus plot for the system is/are

(A) 3 real \hspace{1cm} (B) only real \hspace{1cm} (C) 1 real, 2 complex \hspace{1cm} (D) None

MCQ 1.1.57

The open loop transfer function of a \( \text{ufb} \) system is given below.

\[
G(s)H(s) = \frac{K}{s(s+4)(s+5)}
\]

Consider the following statements for the system.
1. Root locus plot cross \( j\omega \)-axis at \( s = \pm j2\sqrt{5} \) \hspace{1cm} (A) 1 and 2 \hspace{1cm} (B) 1, 2 and 3 \hspace{1cm} (C) 2, 3 and 4 \hspace{1cm} (D) All
2. Gain margin for \( K = 18 \) is 20 dB.
3. Gain margin for \( K = 1800 \) is -20 dB
4. Gain \( K \) at breakaway point is 13.128

Which of the following is correct ?

(A) 1 and 2 \hspace{1cm} (B) 1, 2 and 3 \hspace{1cm} (C) 2, 3 and 4 \hspace{1cm} (D) All
The open loop transfer function of a control system is

\[ G(s)H(s) = \frac{K(s + 1)}{s(s - 1)(s^2 + 4s + 16)} \]

Consider the following statements for the system:
1. Root locus of the system cross \( j\omega \)-axis for \( K = 35.7 \)
2. Root locus of the system cross \( j\omega \)-axis for \( K = 23.3 \)
3. Break away point is \( s = 0.45 \)
4. Break in point is \( s = -2.26 \)

Which of the following statement is correct?
(A) 1, 3 and 4  
(B) 2, 3 and 4  
(C) 3 and 4  
(D) all

*************
EXERCISE 1.2

Common Data For Q. 1 and 2
A root locus of ufb system is shown below.

Common Data For Q. 4 and 5
The root locus for a ufb system is shown below.

QUES 1.2.1
The breakaway point for the root locus of system is ______

QUES 1.2.2
At break-away point, the value of gain $K$ is ______

QUES 1.2.3
The forward-path transfer function of a ufb system is

$$G(s) = \frac{K(s+2)}{(s+3)(s^2+2s+2)}$$

The angle of departure from the complex poles is $\pm \phi_B$; where $\phi_B = ______$ degree.

QUES 1.2.4
The root locus crosses the imaginary axis at $\pm ja$; where $a = ______$

QUES 1.2.5
The value of gain for which the closed-loop transfer function will have a pole on the real axis at $-5$, will be ______
QUES 1.2.6 The open-loop transfer function a system is
\[ G(s)H(s) = \frac{K(s + 8)}{s(s + 4)(s + 12)(s + 20)} \]
A closed loop pole will be located at \( s = -10 \), if the value of \( K \) is ______

QUES 1.2.7 Characteristic equation of a closed-loop system is \( s(s + 1)(s + 2) + K = 0 \). What will be the centroid of the asymptotes in root-locus?

QUES 1.2.8 A unity feedback control system has an open-loop transfer function
\[ G(s) = \frac{K}{s(s^2 + 7s + 12)} \]
The gain \( K \) for which \( s = -1 + j1 \) will lie on the root locus of this system is ______

**Common Data For Q. 9 and 10**
The open loop transfer function of a control system is
\[ G(s)H(s) = \frac{K(s^2 + 2s + 10)}{(s^2 + 6s + 10)} \]

QUES 1.2.9 The angle of departure at the complex poles will be \( \pm \phi_D \); where \( \phi_D = _____ \) degree.

QUES 1.2.10 The angle of arrival at complex zeros is \( \pm \phi_A \); where \( \phi_A = _____ \) degrees.

QUES 1.2.11 The open loop transfer function of a unity feedback system is shown below.
\[ G(s)H(s) = \frac{10K}{(s + 2)(s + 10)} \]
What is the value of \( K \) for which the root locus cross the line of constant damping \( \xi = \frac{1}{\sqrt{2}} \)?

QUES 1.2.12 The root locus plot of a system is shown below.

The gain margin for \( K = 12 \) is ______ dB.
A unity feedback control system has open loop transfer function,

\[ G(s)H(s) = \frac{K}{s(s+10)(s+20)} \]

The value of \( K \) at the breakaway point is ______

The root sensitivity of the system at \( s = -9.47 \) is ______

The open loop transfer function of a system is

\[ G(s)H(s) = \frac{K(s^3 + 4)}{s(s + 2)} \]

The value of \( K \) at breakaway point is ______

**Common Data For Q. 16 and 17**

The root locus plot for a system is given below.

Damping ratio for \( K = 12.2 \) is \( \xi = \) ______

Peak over shoot of the system response for \( K = 12.2 \) is \( M_P = \) ______

**Common Data For Q. 16 and 19**

The block diagram of a control system is given below.

The root locus of the system is plotted as the value of parameter \( \alpha \) is varied. The break away point is \( s = \) ______
QUES 1.2.19 The value of $\alpha$ for which transient response have critical damping, is 

QUES 1.2.20 The open loop transfer function of a uf system is
\[ G(s)H(s) = \frac{K(s+1)}{s^2(s+9)} \]
In the root locus of the system, as parameter $K$ is varied from 0 to $\infty$, the gain $K$ when all three roots are real and equal is 

QUES 1.2.21 The open loop transfer function of a system is
\[ G(s)H(s) = \frac{K}{(s+1)(s+5)} \]
What is the value of $K$, so that the point $s = -3 + j5$ lies on the root locus?
**EXERCISE 1.3**

**MCQ 1.3.1** If the gain \( K \) of a system becomes zero, the roots will
(A) move away from zeros \( \quad \) (B) move away from the poles
(C) coincide with the zeros \( \quad \) (D) coincide with the poles

**MCQ 1.3.2** The root locus plot is symmetrical about the real axis because
(A) roots occur simultaneously in LH and RH planes
(B) complex roots occur in conjugate pairs
(C) all roots occur in pairs
(D) none of these

**MCQ 1.3.3** The break away points of the root locus occur at
(A) imaginary axis
(B) real axis
(C) multiple roots of characteristic equation
(D) either (A) or (B)

**MCQ 1.3.4** The algebraic sum of the angles of the vectors from all poles and zeros to the point on any root locus segment is
(A) 180°
(B) 150°
(C) 180° or its odd multiple
(D) 80° or its odd multiple

**MCQ 1.3.5** The root locus is
(A) an algebraic method \( \quad \) (B) a graphical method
(C) combination of both \( \quad \) (D) none of these

**MCQ 1.3.6** The root locus is a
(A) time-domain approach
(B) frequency domain approach
(C) combination of both
(D) none of these

**MCQ 1.3.7** The root locus can be applied to
(A) only linear systems
(B) only nonlinear systems
(C) both linear and nonlinear systems
(D) none of these
MCQ 1.3.8 The root locus can be used to determine
(A) the absolute stability of a system of a system
(B) the relative stability of a system
(C) both absolute and relative stabilities of a system
(D) none of these

MCQ 1.3.9 The root locus always starts at the
(A) open-loop poles  (B) open-loop zeros
(C) closed-loop poles  (D) closed-loop zeros

MCQ 1.3.10 The root locus always terminates on the
(A) open-loop zeros (B) closed-loop zeros
(C) roots of the characteristic equation  (D) none of these

MCQ 1.3.11 The root locus gives the locus of
(A) open-loop poles
(B) closed-loop poles
(C) both open-loop and closed-loop poles
(D) none of these

MCQ 1.3.12 An open-loop transfer function has 4 poles and 1 zero. The number of
branches of root locus is
(A) 4  (B) 1
(C) 5  (D) 3

MCQ 1.3.13 The open-loop transfer function of a control system has 5 poles and 3 zeros.
The number of asymptotes is equal to
(A) 5  (B) 3
(C) 2  (D) 8

MCQ 1.3.14 Angles of asymptotes are measured at the centroid with respect to
(A) negative real axis  (B) positive real axis
(C) imaginary axis  (D) none of these

MCQ 1.3.15 Break points can be
(A) only real  (B) only complex
(C) real or complex  (D) none of these

MCQ 1.3.16 The angle of departure from a real pole is always
(A) 0°
(B) 180°
(C) either 0° or 180°
(D) to be calculated for each problem
MCQ 1.3.17  The angle of arrival at a real zero is always
(A) 0°
(B) 180°
(C) either 0° or 180°
(D) to be calculated for each problem

MCQ 1.3.18  A unit feedback system has open-loop poles at \( s = -2 \pm j2 \), \( s = -1 \), and \( s = 0 \); and a zero at \( s = -3 \). The angles made by the root-locus asymptotes with the real axis, and the point of intersection of the asymptotes are, respectively,
(A) \( 60°, -60°, 180° \) and \( -3/2 \)
(B) \( 60°, -60°, 180° \) and \( -2/3 \)
(C) \( 45°, -45°, 180° \) and \( -2/3 \)
(D) \( 45°, -45°, 180° \) and \( -4/3 \)

MCQ 1.3.19  Root locus plot of a feedback system as gain \( K \) is varied, is shown in below. The system response to step input is non-oscillatory for

\( K = 0.4 \)
\( K = 6 \)

(A) \( 0 < K < 0.4 \)
(B) \( 0.4 < K < 6 \)
(C) \( 6 < K < \infty \)
(D) None of the answers in (A), (B) and (C) is correct

MCQ 1.3.20  Consider the root locus plot shown in below.

\( K = 0.4 \)
\( K = 6 \)

(P) Adding a zero between \( s = -1 \) and \( s = -2 \) would move the root locus to the left.
(Q) Adding a pole at \( s = 0 \) would move the root locus to the right.
Which of the following is the correct answer?
(A) None of the above statements is true
(B) Statement (i) is true but statement (ii) is false
(C) Statement (i) is false but statement (ii) is true
(D) Both the statements are true

MCQ 1.3.21
Consider the root locus plot of unity-feedback system with open-loop transfer function,
\[ G(s) = \frac{K(s + 5)}{s(s + 2)(s + 4)(s^2 + 2s + 2)} \]
The meeting point of the asymptotes on the real axis occurs at
(A) −1.2
(B) −0.85
(C) −1.05
(D) −0.75

MCQ 1.3.22
The root locus plot of the characteristic equation \(1 + KF(s) = 0\) is given in below. The value of \(K\) at \(s = \pm j1\) is
(A) 4
(B) 1
(C) 10
(D) None of the answers in (A), (B), and (C) is correct.

MCQ 1.3.23
In a root locus plot,
(P) there is only one intersect to the asymptotes and it is always on the real axis;
(Q) the breakaway points always lie on the real axis.
Which of the following is the correct answer?
(A) None of the statements is true
(B) Statement (P) is true but statement (Q) is false
(C) Statement (P) is false but statement (Q) is true
(D) Both the statements are true

MCQ 1.3.24
Consider the following statements:
(P) The effect of compensating pole is to pull the root locus towards left.
(Q) The effect of compensating zero is to press the locus towards right.
Chap 1
Root Locus Technique

MCQ 1.3.25
The loop transfer function \(GH\) of a control system is given by
\[
GH = \frac{K}{s(s+1)(s+2)(s+3)}
\]
Which of the following statements regarding the conditions of the system root loci diagram is/are correct.
1. There will be four asymptotes.
2. There will be three separate root loci.
3. Asymptotes will intersect at real axis at \(\sigma = -2/3\)
Select the correct answer using the codes given below:
Codes:
(A) 1 alone
(B) 2 alone
(C) 3 alone
(D) 1, 2 and 3

MCQ 1.3.26
If the characteristic equation of a closed-loop system is
\[
1 + \frac{K}{s(s+1)(s+2)} = 0
\]
the centroid of the asymptotes in root-locus will be
(A) zero
(B) 2
(C) -1
(D) -2

MCQ 1.3.27
Assertion (A) : The number of separate loci or poles of the closed loop system corresponding to \(G(s)H(s) = \frac{K(s+4)}{s(s+1)(s+3)}\) is three.
Reason (R) : Number of separate loci is equal to number of finite poles of \(G(s)H(s)\) if the latter is more than the number of finite zeros of \(G(s)H(s)\).
(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false
(D) A is false but R is true

MCQ 1.3.28
Which one of the following is correct?
The value of the system gain at any point on a root locus can be obtained as a
(A) product of lengths of vectors from the poles to that point
(B) product of lengths of vectors from the zeros to that point
(C) ratio of product of lengths of vectors from poles to that point to the product of length of vectors from zeros to that point
(D) product of lengths of vectors from all poles to zeros
MCQ 1.3.29 Which one of the following is not a property of root loci?
(A) The root locus is symmetrical about $j\omega$ axis.
(B) They start from the open loop poles and terminate at the open loop zeros.
(C) The breakaway points are determined from $\frac{dk}{ds} = 0$.
(D) Segments of the real axis are part of the root locus, if and only if, the total number of real poles and zeros to their right is odd.

MCQ 1.3.30 The addition of open loop zero pulls the root-loci towards:
(A) The left and therefore system becomes more stable
(B) The right and therefore system becomes unstable
(C) Imaginary axis and therefore system becomes marginally stable
(D) The left and therefore system becomes unstable

MCQ 1.3.31 Which one of the following is correct?
The root locus is the path of the roots of the characteristic equation traced out in the $s$-plane
(A) as the input of the system is changed
(B) as the output of the system is changed
(C) as a system parameter is changed
(D) as the sensitivity is changed

***********
SOLUTIONS 1.1

SOL 1.1.1 Correct option is (D).
We check the validity of root locus for each of the given options.
Option (A):
Root locus is always symmetric about real axis. This condition is not satisfied for option (A). A point on the real axis lies on the root locus if the total number of poles and zeros to the right of this point is odd. This is also not satisfied by (A). Thus, option (A) is not a root locus diagram.
Option (B) & Option (C):
These do not satisfy the condition that a point on the real axis lies on the root locus if the total number of poles and zeros to the right of this point is odd. Thus, option (B) and (C) are also not root locus diagram.
Option (D):
This is symmetric about real axis and every point of locus satisfy the condition that number of poles and zeros in right of any point on locus be odd. Thus, the sketch given in option (D) can be considered as root locus for a system.

SOL 1.1.2 Correct option is (D).
Here, option (2) and option (3) both are not symmetric about real axis. So, both cannot be root locus.

SOL 1.1.3 Correct option is (A).
Here, pole-zero location is given as

The angle of departure of the root locus branch from a complex pole is given by
\[ \phi_D = \pm [180^\circ + \phi] \]
where \( \phi \) is net angle contribution at this pole due to all other poles and zeros, as shown in figure below.
From the pole-zero plot, we have
\[ \phi = \phi_z - \phi_p = [-90° + 90°] - [90° + 90°] = -180° \]
So, the departure angle is
\[ \phi_d = \pm [180° - 180°] = \pm 0° \]
Therefore, the departure angle for pole \( P_0 \) is 0°. Thus, root locus branch will depart at 0°. Only option (A) satisfies this condition.

**SOL 1.1.4**
Correct option is (A).
Given open loop pole-zero plot is

\[ \begin{array}{c}
    j\omega \\
    -X - O - X \\
\end{array} \]

From the given plot, we have
Number of poles, \( P = 2 \)
Number of zeros, \( Z = 1 \)
Since, the number of branches of root locus is equal to number of poles, so we have
Number of branches = 2
Thus (B) and (D) are not correct.
Again, the branch of root locus always starts from open loop pole and ends either at an open loop zero (or) infinite. Thus, (C) is incorrect and remaining Correct option is (A).

**SOL 1.1.5**
Correct option is (C).
Root locus plot starts from poles and ends at zeros (or) infinite. Only option (C) satisfies this condition. No need to check further.

**SOL 1.1.6**
Correct option is (A).
Root locus always starts from open loop pole, and ends at open loop zero (or) infinite. Only option (A) satisfies this condition.
We can find the root locus of given plot as follows
Number of poles, \( P = 2 \)
Number of zeros, \( Z = 0 \)
So, we have number of asymptotes
\[ P - Z = 2 \]
Also, the angle of asymptotes is obtained as
\[ \phi_a = \frac{(2q + 1)180°}{P - Z}; q = 0, 1 \]
\[ \phi_a = \frac{(0 + 1)180°}{2} = 90°; q = 0 \]
and \[ \phi_a = \frac{(2 + 1)180°}{P - Z} = \frac{3 \times 180°}{2} = 270°; q = 1 \]
Hence, we sketch the root locus plot as

![Root Locus Plot]

SOL 1.1.7
Correct option is (A).
An open loop pole-zero plot is given as

![Open Loop Pole-Zero Plot]

Root locus always starts from open loop poles and ends at open loop zeros or infinite along with asymptotes. So, the options (C) and (D) are wrong. Again, a point on the real axis lies on the root locus if the total number of poles and zeros to the right of this point is odd. This is not satisfied by (B). Thus, the remaining correct option is (A).

SOL 1.1.8
Correct option is (C).
Forward path open loop transfer function of given \( \text{ufb} \) system is

\[
G(s) = \frac{K(s+2)(s+6)}{s^2 + 8s + 25}
\]

So, we have the characteristic equation as

\[
s^2 + 8s + 25 = 0
\]

or

\[
s = \frac{-8 \pm \sqrt{64-100}}{2} = -4 \pm j3
\]

i.e. poles of the system are

\[
s = -4 + j3; \ s = -4 - j3
\]

Also, from the given transfer function, we have zeros of the system as

\[
s = -2; \ s = -6
\]

Thus, we get the pole-zero plot as shown below.

![Pole-Zero Plot]

Also, we have the condition that root locus starts from poles and ends with zeros. Only option (C) satisfies this condition.
SOL 1.1.9
Correct option is (B).
Forward path open loop transfer function of given ufb system is
\[ G(s) = \frac{K(s^2 + 4)}{s^2 + 1} \]
So, we obtain the poles of the system as
\[ s^2 + 1 = 0 \]
or
\[ s = \pm j1 \]
Also, zeros of the system are obtained as
\[ s^2 + 4 = 0 \]
or
\[ s = \pm j2 \]
Therefore, the pole-zero plot of the system is

Since, we have the condition that root locus starts from poles and ends with zeros. Thus, Correct option is (B).

SOL 1.1.10
Correct option is (A).
Forward path open loop transfer function of given ufb system is
\[ G(s) = \frac{K(s^2 + 1)}{s^2} \]
So, we obtain the zeros of the system as
\[ s^2 + 1 = 0 \]
or
\[ s = \pm j1 \]
Also, the poles of the system are
\[ s = 0; s = 0 \]
So, we have the pole-zero plot for the system as

Hence, option (B) and (D) may not be correct option. A point on the real axis lies on the root locus if the total number of poles and zeros to the right of this point is odd. This is not satisfied by (C) because at origin there are double pole. Thus, remaining Correct option is (A).

SOL 1.1.11
Correct option is (C).
Given open loop pole zero plot of the system
From above plot, we have the zeros and poles as:

Zeros: \( s = 1 + j1 \) and \( s = 1 - j1 \)

Poles: \( s = -2 \) and \( s = -3 \)

So, the transfer function of the system is obtained as:

\[
G(s) = K \frac{s - (1 + j1)}{s - (-3)} \frac{s - (1 - j1)}{s - (-2)} \]

or

\[
G(s) = \frac{K(s^2 - 2s + 2)}{(s + 2)(s + 3)}
\]

SOL 1.1.12

Correct option is (C).

Root locus lies on real axis where number of poles and zeros are odd in number from that right side.

Hence, for the given pole-zero plot, root locus lies between poles \((-2)\) and \((-3)\) on real axis. From the given option, we have two points \( \sigma = -1.29 \) and \( \sigma = -2.43 \).

Since, \( \sigma = -1.29 \) does not lie on root locus, so it can not be a break point. Therefore, the possible break point is \( \sigma = -2.43 \) which lies between \(-2\) and \(-3\). Now we check whether the point is break away or break in.

On root locus, it may be seen easily that \( \sigma = -2.43 \) lies on root locus and locus start from poles \((-2)\) and \((-3)\). Therefore, at \( \sigma = -2.43 \) it must break apart. Thus, this point is break away point, i.e. the break point is

\[ \text{Break away at } \sigma = -2.43 \]

**ALTERNATIVE METHOD:**

Gain \( K \) will be maximum at break away point and minimum at break in point. We can also check maxima and minima for gain \( K \). The point, at which multiple roots are present, are known as break point. These are obtained from

\[
\frac{dK}{ds} = 0 \quad \ldots(1)
\]

Now, we have the characteristic equation as:

\[ 1 + G(s)H(s) = 0 \]

or

\[ 1 + \frac{K(s^2 - 2s + 2)}{(s + 2)(s + 3)} = 0 \]

So,

\[
K = \frac{-(s + 2)(s + 3)}{(s^2 - 2s + 2)} = \frac{-(s^2 + 5s + 6)}{s^2 - 2s + 2} \quad \ldots(2)
\]

Differentiating equation (2) w.r.t. \( s \) and applying it to equation (1), we have
or \[ 7s^2 + 8s - 22 = 0 \]

Solving the above expressions, we get

\[ s = +1.29 \text{ and } s = -2.43 \]

The point \( s = -2.43 \) is maxima for gain \( K \), so \( s = -2.43 \) is break away point.

**SOL 1.1.13**

Correct option is (A).

Forward path transfer function of given \( \text{ufb} \) system is

\[ G(s) = \frac{K(s+1)(s+2)}{(s+5)(s+6)} \]

So, we have the characteristics equation

\[ 1 + G(s)H(s) = 0 \]

or

\[ 1 + \frac{K(s+1)(s+2)}{(s+5)(s+6)} = 0 \]

or

\[ (s^2 + 11s + 30) + K(s^2 + 3s + 2) = 0 \]

or

\[ K = -\frac{(s^2 + 11s + 30)}{s^2 + 3s + 2} \] \( \ldots (1) \)

Differentiating above equation with respect to \( s \) and equating to zero, we get

\[ \frac{dK}{ds} = \frac{-\left(s^2 + 3s + 2\right)(2s + 11) + \left(s^2 + 11s + 30\right)(2s + 3)}{(s^2 + 3s + 2)^2} = 0 \]

or

\[ 8s^2 + 56s + 68 = 0 \]

or

\[ s = -5.437 \text{ and } s = -1.563 \]

Now, we have the pole-zero plot for the given system as shown below.

\[ j\omega \]

\[ -6 \quad -5 \quad -2 \quad -1 \]

From the diagram, we note that root locus lies on real axis from \(-6\) to \(-5\) and from \(-2\) to \(-1\) because of odd number of pole and zero constrain.

Now, we further note that locus starts from \(-6\) and \(-5\) (poles of the system). Thus, locus must break apart at \( s = -5.437 \), i.e. it is break away point.

Again, the locus end at \(-2\) and \(-1\) (zeros of the system), thus there must be a break in at \( s = -1.563 \).

**SOL 1.1.14**

Correct option is (A).

The given system is shown below.

\[ R(s) \quad [K(s+2)] \quad \frac{(s+1)(s+3)}{\text{C}(s)} \]
We redraw the block diagram after moving take off point as shown below.

So, the forward path transfer function is
\[ G(s) = \frac{-K(s + 2)}{(s + 1)(s + 3)} \]

Root locus is plotted for \( K = 0 \) to \( K = \infty \). But, here the gain \( K \) is negative.
So, we will plot for \( K = -\infty \) to \( K = 0 \). This is called complementary root locus.
For this case, the root locus on the real axis is found to the left of an even count of real poles and real zeros of \( G(s) \). Also, the plot will start from pole and ends on zero. Only option (A) satisfies the condition for given system.

SOL 1.1.15
Correct option is (D).
For given system, forward path transfer function is
\[ G(s) = \frac{K(s + 6)}{(s + 3)(s + 5)} \]

So, the characteristic equation for closed loop transfer function is
\[ 1 + G(s)H(s) = 0 \]
or
\[ 1 + \frac{K(s + 6)}{(s + 3)(s + 5)} = 0 \]
or
\[ K = \frac{-(s^2 + 8s + 15)}{(s + 6)} \]

Differentiating the above expression with respect to \( s \) and equating it to zero, we have
\[ \frac{dK}{ds} = \frac{-(s + 6)(2s + 8) + (s^2 + 8s + 15)}{(s + 6)^2} = 0 \]
or \[ s^2 + 12s + 33 = 0 \]
or \[ s = -7.73 \text{ and } s = -4.27 \]
Thus, we obtain the root locus for the system as shown below.

Observing the root locus, we can easily say that \( s = -4.27 \) is break away point and \( s = -7.73 \) is break in point.

SOL 1.1.16
Correct option is (B).
For the given system, we have the open loop transfer function
\[ G(s)H(s) = \frac{K}{s(s + 1)(s + 2)} \]

In option (A) and option (C), the root locus on the real axis is found to the left of an even count of real poles and real zeros of GH. So, these can not be the root locus diagram. Now, we have the characteristic equation

\[ 1 + G(s)H(s) = 0 \]

or

\[ 1 + \frac{K}{s(s + 1)(s + 2)} = 0 \]

or

\[ s^3 + 3s^2 + 2s + K = 0 \]

So, we have the Routh’s array for the system

\[
\begin{array}{ccc}
 s^3 & 1 & 2 \\
 s^2 & 3 & K \\
 s^1 & \frac{K}{2} \\
 s^0 & K \\
\end{array}
\]

At \( K = 6 \), \( s^1 \) row is zero, thus using auxiliary equation, we get

\[ 3s^2 + 6 = 0 \]

or

\[ s = \pm j\sqrt{2} \]

Root locus cut on \( j\omega \) axis at \( s = \pm j\sqrt{2} \) for \( K = 6 \). Since, the root locus given in option (D) does not cut \( j\omega \) axis. So, it is not the root locus for given system. Therefore, the remaining Correct option is (B).

**SOL 1.1.17**

Correct option is (D).

For given \( ufb \) system, forward transfer function is

\[ G(s) = \frac{K}{s^2} \]

Angle of departure or angle of asymptote for multiple poles is

\[ \phi_a = \frac{(2q + 1)180^\circ}{r} \]

where

\[ r = \text{number of multiple poles} \]
\[ q = 0, 1, 2, \ldots (r - 1) \]

For the given system, we have

\[ r = 2; \ (2 \text{ multiple poles at origin}) \]
\[ q = 0, 1 \]

So, we obtain the angle of departure as

\[ \phi_a = \frac{(0 + 1)180^\circ}{2} = 90^\circ \text{ for } q = 0 \]

and

\[ \phi_a = \frac{(2 + 1)180^\circ}{2} = 270^\circ \text{ for } q = 1 \]

Hence, root locus plot will be as shown below.
Correct option is (C).

For given \( \text{ufb} \) system, open loop transfer function is

\[
G(s) = \frac{K(s + 3)(s + 4)}{(s + 1)(s + 2)}
\]

Given the two points,

\[
s_1 = -2 + j\beta; \quad s_2 = -2 + \frac{1}{\sqrt{2}}
\]

If any point lies on root locus, it satisfies the characteristic equation of the system, i.e.

\[q(s) = 1 + G(s)H(s) = 0\]

or \[|G(s)H(s)| = 1\] (Magnitude)

and \[\angle(G(s)H(s)) = \pm 180^\circ\] (Phase)

Now, we consider the point \( s_1 = -2 + j\beta \) as shown in the diagram below.

We check the point for the phase condition. At \( s = s_1 = -2 + j\beta \), we have

\[
\angle(G(s)H(s)) \bigg|_{s=s_1} = \phi_1 = \theta_1 + \theta_2 + \theta_3 + \theta_4
\]

\[= \tan^{-1} \frac{3}{2} + \tan^{-1} \frac{3}{1} - 90^\circ - \tan^{-1} \left( \frac{3}{1} \right)\]

\[= \tan^{-1} \frac{3}{2} + \tan^{-1} 3 - 90^\circ - (180^\circ - \tan^{-1} 3)\]

\[= 56.30 + 71.56 - 270^\circ + 71.56\]

or

\[\phi_1 = -70.56 \neq \pm 180^\circ\]

Hence, point \( s_1 \) does not lie on root locus. Again, we consider the point \( s_2 = -2 + j\frac{1}{\sqrt{2}} \) as shown in the diagram below.

At the given point \( s_2 \), we have

\[\phi_2 = \theta_1 + \theta_2 - \theta_3 - \theta_4\]

\[= \tan^{-1} \frac{1}{2\sqrt{2}} + \tan^{-1} \frac{1}{\sqrt{2}} - 90^\circ - (180^\circ - \tan^{-1} \frac{1}{\sqrt{2}})\]

\[= \tan^{-1} \frac{1}{2\sqrt{2}} + \tan^{-1} \frac{1}{\sqrt{2}} - 90^\circ - 180^\circ + \tan^{-1} \frac{1}{\sqrt{2}}\]

\[\phi_2 = -180^\circ\]

i.e. phase condition is satisfied. Hence, point \( s_2 \) lies on root locus.
SOL 1.1.19

Correct option is (A).

For given \( ufb \) system, open loop transfer function is

\[
G(s) = \frac{K(s + \alpha)}{s^2(s + \beta)}; \quad \beta > \alpha > 0
\]

So, we have the poles and zeros for the system as

Zero: \( s = -\alpha \)

Poles: \( s = 0, 0, -\beta \)

So, the departure angles at double poles on origin are obtained as

\[
\phi = \left( \frac{2q + 1}{r} \right) 180^\circ; \quad r = 2, q = 0, 1
\]

or

\[
\phi = 90^\circ \text{ and } 270^\circ
\]

To get intersection with imaginary axis, we use Routh’s criteria. The characteristic equation for the system is given as

\[
s^3 + \beta s^2 + Ks + K\alpha = 0
\]

So, we have the Routh’s array as

| \( s^3 \) | 1 | K |
| \( s^2 \) | \( \beta \) | \( K\alpha \) |
| \( s^1 \) | \( \beta - \alpha \) |
| \( s^0 \) | \( K\alpha \) |

Here, for any value of \( K \), \( s^1 \) row of Routh array will not be zero. Thus, system is stable for all positive value of \( K \), and hence root locus does not cross \( j\omega \) axis. Therefore, root locus completely lies in left half of \( s \)-plane.

Based on these results we say that Correct option is (A).

SOL 1.1.20

Correct option is (C).

Characteristic equation of given feedback control system is

\[
(s^2 + 4s + 4)(s^2 + 11s + 30) + Ks^2 + 4K = 0
\]

or

\[
1 + \frac{K(s^2 + 4)}{(s^2 + 4s + 4)(s^2 + 11s + 30)} = 0 \quad \ldots (1)
\]

Since, the characteristic equation of a system is defined as

\[
1 + G(s)H(s) = 0 \quad \ldots (2)
\]

Comparing equations (1) and (2), we get open loop transfer function as

\[
G(s)H(s) = \frac{K(s^2 + 4)}{(s^2 + 4s + 4)(s^2 + 11s + 30)} = \frac{K(s^2 + 4)}{(s+2)(s+2)(s+5)(s+6)}
\]

So, the open loop poles and zeros of the system are

Poles: \( s = -2, -2 \) and \( s = -6, -5 \)

Zeros: \( s = \pm j2 \)

The point at which asymptotes meet (centroid) is given by

\[
\sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{(P - Z)}
\]

\[
= \frac{(-2 - 2 - 5 - 6) - 0}{4 - 2} = \frac{-15}{2} = -7.5
\]

This is the point on real axis. So, the asymptotes meet at \((-7.5, 0)\)
SOL 1.1.21
Correct option is (D).
Open loop transfer function for given system is
\[ G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+5)} \]
So, we have the characteristic equation
\[ 1 + \frac{K}{s(s+4)(s^2+4s+5)} = 0 \]
or
\[ K = -s(s+4)(s^2+4s+5) \]  \[... (1)\]
Differentiating the above equation with respect to \( s \) and equating it to zero, we have
\[ \frac{dK}{ds} = -[(2s+4)(s^2+4s+5)+(s^2+4s)(2s+4)] = 0 \]
or
\[ -(2s+4)(s^2+4s+5+s^2+4s) = 0 \]
or
\[ -(s+2)(2s^2+8s+5) = 0 \]  \[... (2)\]
Solving the above equation, we get
\[ s = -2 \text{ and } s = -0.775, -3.225 \]
Now, we check for maxima and minima value of gain \( K \) at above point. If gain is maximum, then that point will be break away point. If gain is minimum, then that point will be break in point. Again, differentiating equation (2) with respect to \( s \), we get
\[ \frac{d^2K}{ds^2} = -[(2s^2+8s+5)+(s+2)(4s+8)] \]
\[ = -(6s^2+24s+21) \]
For \( s = -0.775 \) and \( s = -3.225 \), we have
\[ \frac{d^2K}{ds^2} = -6.0 < 0 \]
So, the points \( s = -0.775 \) and \( s = -3.225 \) are maxima points. Hence, \( s = -0.775 \) and \( s = -3.225 \) are break away points. Again, for \( s = -2 \), we have
\[ \frac{d^2K}{ds^2} = +3 > 0 \]
So, the point \( s = -2 \) is minima points. Hence, \( s = -2 \) is break in point. Thus, there is two break away points \( (s = -0.775, -3.225) \) and one break in point \( (s = -2) \).

SOL 1.1.22
Correct option is (B).
For a \( ufb \) system, forward transfer function is
\[ G(s) = \frac{1}{s(s+\alpha)} \]
So, the characteristic equation of system is obtained as
\[ 1 + G(s)H(s) = 0 \]
or
\[ 1 + \frac{1}{s(s+\alpha)} = 0 \]
or
\[ s^2 + \alpha s + 1 = 0 \]
or
\[ 1 + \frac{\alpha s}{(s^2+1)} = 0 \]
Open loop transfer function, as \( \alpha \) is varied, is
\[ G(s)H(s) = \frac{\alpha s}{s^2+1} \]
So, we have the open loop poles and zeros as
zero : \( s = 0 \)
and \[ s^2 + 1 = 0 \Rightarrow s = \pm j \]

Therefore, we sketch the root locus for the system as

![Root Locus Diagram](image)

**ALTERNATIVE METHOD:**

The closed loop transfer function for the system is

\[ T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{1}{s^2 + \alpha s + 1} \]

So,

\[ \frac{G(s)}{1 + G(s)H(s)} = \frac{1}{s^2 + \alpha s + 1} \]

or

\[ G(s)H(s) = \frac{\alpha s}{s^2 + 1} \]

Now, we sketch the root locus by following the steps described in previous method.

**SOL 1.1.23**

Correct option is (A).

Forward path transfer function of the given \( ufb \) system is

\[ G(s) = \frac{K(s + \alpha)(s + 3)}{s(s^2 - 1)}; \alpha = 5 \text{ and } K > 0 \]

or

\[ G(s) = \frac{K(s + 5)(s + 3)}{s(s^2 - 1)} \]

So, we have the open loop poles and zeros for the system as

Zeros: \( s = -5, s = -3 \)

Poles: \( s = 0, s = 1, s = -1 \)

Locus branches start from poles and ends on zeros or infinite along asymptote. Here, number of asymptotes is

\[ P - Z = 3 - 2 = 1 \]

Observing all the given options, we conclude that only option (A) has one asymptote. Now, the angle of asymptotes is given as

\[ \phi_q = \frac{(2q + 1)180°}{P - Z}; q = 0, 1, 2, ..., (P - Z - 1) \]

\[ = \frac{(0 + 1)180°}{1} = 180° \]

Only option (A) satisfies these conditions.

**SOL 1.1.24**

Correct option is (C).

Forward path transfer function of the given system is

\[ G(s) = \frac{K(s + \alpha)(s + 3)}{s(s^2 - 1)}; K = 10 \text{ and } \alpha > 0 \]
So, the characteristic equation of the system is obtained as

\[ 1 + G(s)H(s) = 0 \]

or

\[ 1 + \frac{10(s+\alpha)(s+3)}{s(s^2-1)} = 0 \]

or

\[ s^3 - s + 10[s^2 + (\alpha + 3)s + 3\alpha] = 0 \]

or

\[ s(s^2 + 10s + 29) + \alpha 10(s + 3) = 0 \] ... (1)

or

\[ 1 + \frac{\alpha 10(s + 3)}{s(s^2 + 10s + 29)} = 0 \] ... (2)

So, we have the open loop gain as \( \alpha \) is varied,

\[ G(s)H(s) = \frac{\alpha 10(s + 3)}{s(s^2 + 10s + 29)} \] ... (3)

Therefore, the number of asymptotes are

\[ P - Z = 3 - 1 = 2 \]

So, two root branches will go to infinite along asymptotes as \( \alpha \to \infty \). Now, from equation (1) we have

\[ s^3 + 10s^2 + (29 + 10\alpha)s + 30\alpha = 0 \]

So, we form the Routh’s array as

<table>
<thead>
<tr>
<th>( s^3 )</th>
<th>1</th>
<th>29 + 10\alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^2 )</td>
<td>10</td>
<td>30\alpha</td>
</tr>
<tr>
<td>( s^1 )</td>
<td>29 + 7\alpha</td>
<td></td>
</tr>
<tr>
<td>( s^0 )</td>
<td>30\alpha</td>
<td></td>
</tr>
</tbody>
</table>

For \( \alpha > 0 \), \( s^3 \) row can not be zero. Hence, root locus does not intersect \( j\omega \) axis for \( \alpha > 0 \). Only option (C) satisfies these conditions.

**SOL 1.1.25**

Correct option is (B).

Given the open loop transfer function,

\[ G(s)H(s) = \frac{K(s + 6)}{(s + 2)(s + 4)} \]

So, we have the open loop poles and zeros as

- Poles : \( s = -2 \) and \( s = -4 \)
- Zeros : \( s = -6 \)

Therefore, the number of asymptotes is

\[ P - Z = 2 - 1 = 1 \]

So, the characteristic (1) is correct.

Now, we have the characteristic equation for the system

\[ (s + 2)(s + 4) + K(s + 6) = 0 \]

or

\[ s^2 + (6 + K)s + 8 + 6K = 0 \]

For the characteristic equation, we form the Routh’s array as

<table>
<thead>
<tr>
<th>( s^2 )</th>
<th>1</th>
<th>8 + 6K</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^1 )</td>
<td>6 + K</td>
<td></td>
</tr>
<tr>
<td>( s^0 )</td>
<td>8 + 6K</td>
<td></td>
</tr>
</tbody>
</table>

Root locus is plotted for \( K = 0 \) to \( \infty \), i.e. \( K > 0 \).
Here, for $K > 0$ root locus does not intersect $j\omega$ axis because $s^3$ row will not be zero. Thus, characteristic (2) is incorrect.

For the given system, we have two poles and one zero. So, one imaginary zero lies on infinite. Therefore, the characteristic (4) is incorrect.

Hence, (B) must be correct option. But, we check further for characteristic (3) as follows. We sketch the root locus for given system as

![Root Locus Diagram](attachment:root_locus.png)

It has two real axis intersections. So, characteristic (3) is correct.

**SOL 1.1.26**

Correct option is (C).

Forward path transfer function of given system is

$$G(s) = \frac{K(s + 3)}{s(s + 1)(s + 2)(s + 4)}$$

So, the open loop poles and zeros are zero: $s = -3$ (i.e. $Z = 1$)
and poles: $s = 0$, $s = -1$, $s = -2$, $s = -4$ (i.e. $P = 4$)

So, we obtain the angle of asymptotes as

$$\phi_a = \frac{(2q + 1)180^\circ}{P - Z}; \quad q = 0, 1, 2, ... (P - Z - 1)$$

$$\phi_a = \frac{(0 + 1)180^\circ}{4 - 1} = \frac{180^\circ}{3} = 60^\circ; \quad q = 0$$
$$\phi_a = \frac{(2 + 1)180^\circ}{4 - 1} = 180^\circ; \quad q = 1$$
$$\phi_a = \frac{(4 + 1)180^\circ}{4 - 1} = 300^\circ; \quad q = 2$$

Thus, $\phi_a = \left\{ \begin{array}{ll} 60^\circ = \frac{\pi}{3}; & q = 0 \\ 180^\circ = \pi; & q = 1 \\ 300^\circ = \frac{5\pi}{3}; & q = 2 \end{array} \right.$

**SOL 1.1.27**

Correct option is (B).

For the given system, we have the open loop transfer function

$$G(s)H(s) = \frac{K(s + 10)(s^2 + 20s + 500)}{s(s + 20)(s + 50)(s^2 + 4s + 5)}$$

For the open loop transfer function, we obtain

Separate loci = Number of open loop poles = 5
Asymptotes = Number of OLP – Number of OLZ = 5 – 3 = 2

Matching the two parameters of root locus, we say that Correct option is (B). But, we check further for other characteristic as follows.

Loci on real axis = number of poles that lie on real axis
$$= 3; \quad (s = 0, s = -20, s = -50)$$

Also, we have the open loop poles and zeros for the system as
zeros: $s = -10, s = -10 \pm j20$
poles: \( s = 0, \ s = -20, \ s = -50, \ s = -2 \pm j1 \)

So, we have the pole zero plot as shown below.

![Pole Zero Plot]

So, we obtain the centroid as

\[
\sigma_A = \frac{(0 - 20 - 50 - 2 - 2) - (-10 - 10 - 10)}{5 - 3} = -22
\]

Also, the angle of asymptotes is given as

\[
\phi_a = \frac{(2q + 1)180^\circ}{P - Z}; \quad P - Z = 2, \ q = 0, 1
\]

\[
= \frac{(0 + 1)180^\circ}{2} = 90^\circ; \quad q = 0
\]

\[
= \frac{(2 + 1)180^\circ}{2} = 270^\circ; \quad q = 1
\]

Therefore, we get the root locus for the system as

![Root Locus]

Here, root locus lies only in the region on real axis that is in left of an odd count of real poles and real zeros.

Hence, root locus lies between \(-20\) and \(-50\) and break away point will also be in this region. Thus, there will be only one break away point.

**SOL 1.1.28**

Correct option is (B).

The loci starts from \( s = -1 \) and 0, and ends at \( s = -3 \) and \( \infty \). Hence, poles are \(-1, 0\), and zeros are \(-3, \infty\). Thus, the transfer function of the system is

\[
\frac{K(s + 3)}{s(s + 1)}
\]

**SOL 1.1.29**

Correct option is (D).

The characteristic equation of the given system is

\[
s^2 + 5Ks + 9 = 0
\]
or
\[ 1 + \frac{K_5 s}{s^2 + 9} = 0 \]

Since, we defined the characteristic equation as
\[ 1 + G(s)H(s) = 0 \]

So, open loop transfer function of the system is
\[ G(s)H(s) = \frac{5Ks}{s^2 + 9} \]

Therefore, we have the open loop poles and zeros of the system,
poles: \( s = \pm j3 \)
zeros: \( s = 0 \)

Option (A) and option (B) are incorrect because root locus are starting from zeros. On real axis, loci exist to the left of odd number of real poles and real zeros. Hence, only Correct option is (D).

**SOL 1.1.30**

Correct option is (C).

Open loop transfer function of given \( ufb \) system is
\[ G(s) = \frac{K(1-s)}{s(s+3)} = \frac{-K(s-1)}{s(s+3)} \]

So, we have the open loop poles and zeros as
poles: \( s = 0, s = -3 \)
zeros: \( s = 1 \)

Here, gain \( K \) is negative, so root locus will be complementary root locus and is found to the left of an even count of real poles and real zeros of \( GH \).

Hence, option (A) and option (D) are incorrect. Option (B) is also incorrect because it does not satisfy this condition. Thus, option (C) gives the correct root locus diagram.

**SOL 1.1.31**

Correct option is (A).

Open loop transfer function of the given system is
\[ G(s) = \frac{K(s + \frac{3}{2})}{s^2(s + 2)} \]

So, we have the open loop poles and zeros as
poles: \( s = 0, s = 0, s = -2 \)
zero: \( s = -\frac{3}{2} \)

Therefore, the number of asymptotes is given as
\[ P - Z = \text{Number of OLP} - \text{Number of OLZ} \]
\[ = 3 - 1 = 2 \]

So, we obtain the angle of asymptotes
\[ \phi_a = \frac{(2q+1)180^\circ}{P - Z}; P - Z = 2, q = 0, 1 \]

or
\[ \phi_a = \frac{(0+1)180^\circ}{2} = 90^\circ \text{ for } q = 0 \]

and
\[ \phi_a = \frac{(2+1)180^\circ}{2} = 270^\circ \text{ for } q = 1 \]

The centroid is obtained as
\[ \sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P - Z} \]
From root locus, it can be observed easily that for all values of gain $K$ ($K = 0$ to $\infty$) root locus lie only in left half of $s$-plane.

SOL 1.1.32

Correct option is (C).

Characteristic equation of the given closed loop system is

$$s(s + 1)(s + 3) + K(s + 2) = 0; \quad K > 0$$

or

$$1 + \frac{K(s + 2)}{s(s + 1)(s + 3)} = 0$$

So, the open loop transfer function is given as

$$G(s)H(s) = \frac{K(s + 2)}{s(s + 1)(s + 3)}$$

Therefore, we have the open loop poles and zeros as

- poles: $s = 0$, $s = -1$, $s = -3$
- zero: $s = -2$

So, we obtain the pole zero plot for the system as

For the pole-zero location, we obtain the following characteristic of root locus

- Number of asymptotes: $P - Z = 3 - 1 = 2$
- Angles of asymptotes: $\phi_a = \frac{(2q + 1)180^\circ}{P - Z}; \quad P - Z = 2, \quad q = 0, 1$
  - $\phi_a = 90^\circ$ and $270^\circ$
- Centroid:
  $$\sigma_4 = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P - Z}$$
\[
\frac{0 - 1 - 3 - (-2)}{3 - 1} = \frac{-2}{2} = -1
\]

Thus, from above analysis, we sketch the root locus as

For the root locus, we conclude the following points
1. The break away point lies in the range,
   \[-1 < \text{Re}[s] < 0\]
2. Two of its roots tends to infinite along the asymptotes \(\text{Re}[s] = -1\).
3. Root locus lies only in left half of \(s\)-plane.

**SOL 1.1.33**

Correct option is (B).

For closed loop system, given characteristic equation is

\[(s^2 - 4)(s + 1) + K(s - 1) = 0\]  \(\ldots(1)\)

or \[1 + \frac{K(s - 1)}{(s^2 - 4)(s + 1)} = 0\] \(\ldots(2)\)

Since, we define the characteristic equation as

\[1 + G(s)H(s) = 0\]

So, the open loop transfer function of the system is obtained as

\[G(s)H(s) = \frac{K(s - 1)}{(s^2 - 4)(s + 1)}\]

For the given system, we have the open loop poles and zeros as

- poles: \(s = 2, s = -2, s = -1\)
- zeros: \(s = 1\)

So, we obtain the following characteristic for root locus

Number of branches of loci: \(P = \text{number of OLP}\)
\[= 3\]

Number of asymptotes: \(P - Z = \text{number of OLP} - \text{number of OLZ}\)
\[= 3 - 1 = 2\]

Angle of asymptotes:
\[\phi_a = \frac{(2q + 1)180^\circ}{P - Z};\quad P - Z = 2, q = 0, 1\]
\[\phi_a = \frac{(0 + 1)180^\circ}{2} = 90^\circ\text{ for }q = 0\]
\[\phi_a = \frac{(2 + 1)180^\circ}{2} = 270^\circ\text{ for }q = 1\]

Centroid:
\[\sigma_a = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P - Z}\]
\[= \frac{(-1 - 2 + 2) - (1)}{3 - 1} = \frac{-2}{2} = -1\]
Root locus on the real axis is found to the left of an odd count of real poles and zeros of $GH$. From equation (2), we have

$$K = -\frac{(s^2 - 4)(s + 1)}{(s - 1)}$$

Now, we obtain the break-away point (point of maxima) as

$$\frac{dK}{ds} = 0$$

or

$$-(s - 1)[(s^2 - 4) + (s + 1)(2s)] + (s^2 - 4)(s + 1) = 0$$

or

$$s = -1.5$$

This is a break away point. From above analysis, we sketch the root locus as

![Root Locus Plot](image)

**SOL 1.1.34** Correct option is (A).

Given root locus plot,

![Root Locus Plot](image)

From given plot, we can observe that centroid (point where asymptotes intersect on real axis) is origin and all three root locus branches also start from origin and goes to infinite along with asymptotes. Therefore, there is no any zero and three poles are at origin.

So, option (A) must be correct.

$$G(s) = \frac{K}{s^3}$$

Now, we verify the above result as follows. Using phase condition, we have

$$\left.\frac{G(s)H(s)}{s}\right|_{s=-s} = \pm 180^\circ$$

From given plot, for a given point on root locus, we have

$$\left.\frac{G(s)H(s)}{s}\right|_{s=(1,\sqrt{3})} = -3\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

$$= -3\tan^{-1}(\sqrt{3})$$

$$= -3 \times 60^\circ = -180^\circ$$
Correct option is (B).

Given open loop transfer function,

\[ G(s) = \frac{K}{s^2} \]

So, we have the open loop poles \( s = 0, 0 \); i.e.

Number of poles, \( P = 2 \)

Number of zeros, \( Z = 0 \)

Therefore, root loci starts \( (K = 0) \) from \( s = 0 \) and \( s = 0 \). Since, there is no open loop zero, root loci terminate \( (K = \infty) \) at infinity. Now, we obtain the characteristics of root locus as

Angle of asymptotes:

\[ \phi_a = \frac{(2q + 1)180^\circ}{P - Z}; \quad P - Z = 2 - 0 = 2, \quad q = 0, 1 \]

\[ = \frac{(0 + 1)180^\circ}{2} = 90^\circ \text{ for } q = 0 \]

\[ = \frac{(2 + 1)180^\circ}{2} = 270^\circ \text{ for } q = 1 \]

Centroid:

\[ \sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P - Z} = \frac{0 - 0}{2} = 0 \]

Break-away point:

\[ 1 + \frac{K}{s^2} = 0 \]

or

\[ K = -s^2 \]

So,

\[ \frac{dK}{ds} = -2s = 0 \]

Thus, from the above analysis, we have the root locus plot as

---

Correct option is (B).

Open loop transfer function of given \( ufb \) system is

\[ G(s) = \frac{2(s + \alpha)}{s(s + 2)(s + 10)} \]

So, we have the characteristic equation as

\[ 1 + \frac{2(s + \alpha)}{s(s + 2)(s + 10)} = 0 \]

or

\[ s(s + 2)(s + 10) + 2s + 2\alpha = 0 \]

or

\[ s^3 + 12s^2 + 22s + 2\alpha = 0 \]

or

\[ 1 + \frac{2\alpha}{s^3 + 2s^2 + 22s} = 0 \]

Therefore, we get the open loop transfer function as \( \alpha \) varies,

\[ G(s)H(s) = \frac{2\alpha}{s^3 + 12s^2 + 22s} \]
So, the number of open loop poles and zeros are

Number of poles, \( P = 3 \)
Number of zeros, \( Z = 0 \)

Also, we obtain the angle of asymptotes as

\[
\phi_a = \frac{(2q+1)180^\circ}{P-Z}; \ P - Z = 3; \ q = 0, 1, 2
\]
\[
= \frac{(0+1)180^\circ}{3} = 60^\circ \text{ for } q = 0
\]
\[
= \frac{(2+1)180^\circ}{3} = 180^\circ \text{ for } q = 1
\]
\[
= \frac{(4+1)180^\circ}{3} = 300^\circ \text{ for } q = 2
\]

\( \phi_a = 60^\circ, 180^\circ, 300^\circ \)

SOL 1.1.37

Correct option is (C).

The intercept point (centroid) of asymptotes is defined as

\[
\sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P - Z}
\]

Since, we have the open loop poles and zeros as

Poles: \( s = 0, s = -2, s = -10 \)
Zeros: No any zero

Therefore, we get

\[
\sigma_A = \frac{(0 - 2 - 10) - 0}{3 - 0} = -4
\]

SOL 1.1.38

Correct option is (C).

For the given system, we have the open loop transfer function

\[
G(s)H(s) = \frac{2\alpha}{s^3 + 12s^2 + 22s}
\]

So, we obtain the gain \((K)\) for the system as

\[
K = -\frac{(s^3 + 12s^2 + 22s)}{2}
\]

For break-away point (maxima point), we have

\[
\frac{dK}{ds} = 0
\]
\[
\frac{-(3s^2 + 24s + 22)}{2} = 0
\]

or \( -3s^2 - 24s - 22 = 0 \)

So, \( s = -1.056, -6.943 \)

Thus, the break-away points are \( s = -1.056 \) and \(-6.943\)

SOL 1.1.39

Correct option is (A).

The characteristics equation of given system is

\[
s^3 + 2s^2 + Ks + K = 0
\]

or \( s^3 + 2s^2 + K(s + 1) = 0 \)

or \( 1 + \frac{K(s + 1)}{s^2(s + 2)} = 0 \)
So, we have the open loop transfer function
\[ G(s)H(s) = \frac{K(s+1)}{s^2(s+2)} \]

For the system, we have open loop poles and zeros
- zeros: \( s = -1 \); Number of zeros: \( Z = 1 \)
- poles: \( s = 0, s = 0, s = -2 \); Number of poles: \( P = 3 \)

Root loci starts \((K = 0)\) at \( s = 0, s = 0 \) and \( s = -2 \). One of root loci terminates at \( s = -1 \) and other two terminates at infinity. So, we have the characteristic of root loci as given below.

Number of asymptotes:
\[ P - Z = 2 \]

Angle of asymptotes:
\[ \phi_a = \frac{(2q + 1)180^\circ}{P - Z}; P - Z = 2, q = 0, 1 \]
\[ = \frac{(0 + 1)180^\circ}{2} = 90^\circ; q = 0 \]
\[ = \frac{(2 + 1)180^\circ}{2} = 270^\circ; q = 1 \]

Intercept point (centroid) of asymptotes on real axis:
\[ \sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P - Z} \]
\[ = \frac{0 + 0 - 2 - (-1)}{3 - 1} = -\frac{1}{2} = -0.5 \]

So, we get the root locus for the system as

Correct option is \((C)\).
From the root locus plot, we can observe that the difference between the values of \( K \) at the break point and at the point of intersection of the root locus with the imaginary axis gives the range of \( K \). Since, the poles are complex conjugate in this region, so the system has damped oscillatory response. Hence, we first find value of \( K \) for the point of intersection with imaginary axis and then determine value of \( K \) at the break away point. For the given system, we have the Routh’s array as

\[
\begin{array}{cccc}
\text{s}^3 & 1 & 8 \\
\text{s}^2 & 6 & K \\
\text{s} & 48 - K \\
\text{s}^0 & K \\
\end{array}
\]
The characteristic equation is given by
\[ 1 + G(s)H(s) = 0 \]
or
\[ \frac{1 + \frac{K}{s(s + 2)(s + 4)}}{s^3 + 6s^2 + 8s + K} = 0 \]
For intersection of root locus with imaginary axis, \( s^1 \) row should be zero, i.e.
\[ \frac{48 - K}{6} = 0 \]
or
\[ K = 48 \]
The breakaway point is given by
\[ \frac{dK}{ds} = 0 \]
...(2)
From equation (1), we have
\[ K = -(s^3 + 6s^2 + 8s) \]
Substituting it in equation (2), we get
\[ \frac{dK}{ds} = -(3s^2 + 12s + 8) = 0 \]
or
\[ 3s^2 + 12s + 8 = 0 \]
or
\[ s = -2 \pm 1.15 \]
So,
\[ s = -3.15 \text{ and } s = -0.85 \]
From the plot, we can observe that \( s = -0.85 \) is the actual break away point out of these two points. For \( s = -0.85 \), the value of \( K \) is obtained as
\[ K = \frac{\Pi(\text{Phasor lengths from } s_0 \text{ to the OLP})}{\Pi(\text{Phasor lengths form } s_0 \text{ to the OLZ})} \]
Here, no single zero in the system, hence
\[ K = 0.85 \times 1.15 \times 3.15 = 3.08 \]
Therefore, the range of \( K \) for damped oscillatory system is
\[ 3.08 < K < 48 \]

SOL 1.1.42
Correct option is (D).
1. From given root locus plot, we can see that for all positive values of \( K \) (\( K = 0 \) to \( \infty \)) system poles lie in left half of \( s \)-plane, hence system is stable for all positive values of \( K \).
2. Only for \( K = 7.464 \) and \( K = 0.573 \), poles of the system are real and repeated. For range of \( K : 0.573 < K < 7.464 \) poles of the system are complex conjugate in LH of \( s \)-plane.
3. System has damped oscillatory response for complex conjugate poles in left half which is possible only for \( 0.573 < K < 7.464 \).
4. When the system has real and distinct poles, then response is overdamped. Hence, from root locus plot, real and distinct poles are possible for \( 0 < K < 0.573 \) and also for \( K > 7.464 \).

SOL 1.1.43
Correct option is (B).
For low frequencies, we have
\[ e^{-s} \approx 1 - s \]
So, the open loop transfer function is
\[ G(s)H(s) = \frac{K(1 - s)}{s(s + 2)} = \frac{-K(s - 1)}{s(s + 2)} \]
or \[ |G(s)H(s)| = \left| \frac{K(1 - s)}{s(s + 2)} \right| = 1 \]

or

\[ K = \frac{s(s + 2)}{1 - s} \]

The break points are given by the solution of

\[ \frac{dK}{ds} = 0 \]

or

\[ \frac{d}{ds} \left[ \frac{s(s + 2)}{1 - s} \right] = 0 \]

or \((1 - s)(2s + 2) - s(s + 2)(-1) = 0\)

or

\[ s^2 - 2s - 2 = 0 \]

Therefore, the break points are

\[ s = \pm \frac{2 \pm \sqrt{4 + 8}}{2} = 1 \pm \sqrt{3} \]

\[ = 2.73, -0.73 \text{ or } -0.73 \]

Since, \(G(s)H(s)\) is negative, so the root locus will be complementary root locus and will exist at any point on the real axis, if the total number of poles and zeros to the right of that point is even.

Root locus will exist on real axis between \(s = -2\) and 0 and also for \(s > 1\) . Hence, break away point will be \(s = -0.73\) and break in point will be \(s = +2.73\)

\[ |G(s)H(s)| = \left| \frac{K}{s(s + 4)(s^2 + 4s + 20)} \right| = 1 \]

or

\[ K = \frac{s(s + 4)(s^2 + 4s + 20)}{s + 4} \]

The break points are given by the solution of

\[ \frac{dK}{ds} = 0 \]

or

\[ -(4s^3 + 24s^2 + 72s + 80) = 0 \]
or \( s^3 + 6s^2 + 18s + 20 = 0 \)

or \( (s + 2)(s^2 + 4s + 10) = 0 \)

Hence, solving the above equation, we get the break points as

\[ s = -2 \quad \text{and} \quad s = -2 \pm j2.45 \]

i.e. the root locus has one real break point and two complex break point.

**ALTERNATIVE METHOD:**

For the open loop transfer function of the form,

\[ G(s)H(s) = \frac{K}{s(s + a)(s^2 + bs + c)} \]

if \( \left( -\frac{a}{2} \right) = \left( -\frac{b}{2} \right) \)

then, number of break points = 3

These may be all three real (1 real + 2 real).

If \( \left( -\frac{a}{2} \right) \neq \left( -\frac{b}{2} \right) \)

then, number of break points = 1 real

if \( \left( -\frac{a}{2} \right) = \left( -\frac{b}{2} \right) \)

then, we check

\[ \left| -\frac{a}{2} \right| \times x = c \]

Here, if \( x \leq 5 \Rightarrow 3 \) real break point

and if \( x > 5 \Rightarrow 1 \) real and 2 complex

For given problem, we have

\[ G(s)H(s) = \frac{K}{s(s + a)(s^2 + bs + c)} \]

\[ = \frac{K}{s(s + 4)(s^2 + 4s + 20)} \]

Here, we have

\[ a = 4, \quad b = 4, \quad c = 20 \]

So,

\[ -\frac{a}{2} = -2 = -\frac{b}{2} \]

Hence, there are three break points.

and \[ \left| -\frac{a}{2} \right| \times x = c \]

or \[ \left| (-2) \right| \times x = 20 \Rightarrow x = 10 > 5 \]

Thus, there are 1 real and 2 complex break points.

**SOL 1.1.44**

Correct option is (A).

The open loop transfer function is

\[ G(s)H(s) = \frac{Ks}{s^2 - s + 4.25} \]

Here, \[ \left| G(s)H(s) \right| = \frac{Ks}{s^2 - s + 4.25} = 1 \]

or \[ K = \frac{s^2 - s + 4.25}{s} \]

The break point is given by the solution of \[ \frac{dK}{ds} = 0 \]

or \[ \frac{d}{ds} \left[ \frac{s^2 - s + 4.25}{s} \right] = 0 \]
or \[ s^2 - 4.25 = 0 \]
or \[ s^2 = 4.25 \]
So, \[ s = \pm 2.06 \]
The critical point, \( s = -2.06 \) is break point that belongs to the root locus. 
The other critical point \( s = 2.06 \) belongs to the complementary root locus. 
From the given gain \( (K) \) plot, we can see that at point \( s = -2.06 \), gain has a minima. Hence, this point will be break in point. Because, minimum value of gain \( K \) is achieved at break in point and maximum value at break away point.

**SOL 1.1.45**
Correct option is (B).
The characteristic equation is given by
\[ 1 + G(s)H(s) = 0 \]
or \[ 1 + \frac{K(s-1)}{(s+1)(s+2)} = 0 \]
or \[ s^2 + (3 + K)s + (2 - K) = 0 \]  
...(1)
It is required that one pole should lie at \( s = 0 \). Let another pole lies at \( s = -P \), then required equation is
\[ (s + 0)(s + P) = 0 \]
or \[ s^2 + Ps = 0 \]  
...(2)
On comparing equations (1) and (2), we get
\[ 2 - K = 0 \]
or \[ K = 2 \]

**SOL 1.1.46**
Correct option is (B).
Substituting \( K = 2 \) in equation (1) in previous solution, we have
\[ s^2 + 5s = 0 \]  
...(1)
It is required that one pole at \( s = 0 \) and other pole at \( s = -P \). So, the required equation is
\[ s(s + P) = 0 \]
\[ s^2 + Ps = 0 \]  
...(2)
Comparing equations (1) and (2), we get
\[ P = 5 \]
Hence, other pole is located at \((-5, 0)\).

**SOL 1.1.47**
Correct option is (C).
The characteristic equation of the system is given by
\[ 1 + G(s)H(s) = 0 \]
or \[ 1 + \frac{K(s+2)}{(s+1)(s+3)} = 0 \]
or \[ s(s + 1)(s + 30) + K(s + 2) = 0 \]
or \[ s^3 + 31s^2 + (30 + K)s + 2K = 0 \]  
...(1)
We require the two poles with real part of \(-2\), i.e.
\[ s = -2 \pm j\omega \]
Assume other pole is \( s = -P \), then required equation is
\[ (s + P)(s + 2 + j\omega)(s + 2 - j\omega) = 0 \]
or \( (s + P)(s^2 + 4s + 4 + \omega^2) = 0 \) 
\( s^3 + (4 + P)s^2 + (4 + 4P + \omega^2)s + P(4 + \omega^2) = 0 \) \( \ldots (2) \)

Now, comparing equations (1) and (2), we have
\[ P(4 + \omega^2) = 2K \] \( \ldots (3) \)
and \( (4 + 4P + \omega^2) = 30 + K \) \( \ldots (4) \)
and \( (4 + P) = 31 \Rightarrow P = 27 \) \( \ldots (5) \)

Solving equations (3), (4), and (5), we get
\[ 27(4 + \omega^2) = 2K \]
or \[ (112 + \omega^2) = 30 + K \]
or \[ \omega^2 = \frac{2K}{27} - 4 = (30 + K - 112) \]
or \[ 2K - 108 = 27K - 2214 \]
or \[ 25K = 2106 \]
So, \[ K = \frac{2106}{25} = 84.24 \]

**SOL 1.1.48**
Correct option is (C).
From previous solution, we have third pole at \( s = -P \)
Since, \( P = 27 \)
Hence, third pole of closed loop system is at \( s = -27 \)

**SOL 1.1.49**
Correct option is (C).
From previous solution, we have \( P = 27 \) and \( K = \frac{2106}{25} \)
Then, from equation (3) in above solution, we have \[ \omega^2 = \frac{2K}{27} - 4 = 2.24 \]
or \[ \omega = \pm 1.497 \]
Therefore, complex poles are
\[ s = -2 \pm j\omega \]
\[ = -2 \pm j1.497 \]

**SOL 1.1.50**
Correct option is (C).
For delay time \( \tau_D = 1 \text{ sec} \), the characteristic equation of the system is
\[ 1 + \frac{Ke^{-s}}{s} = 0; \ K \geq 0 \]
Now, we have the approximation
\[ e^{-s} \approx \frac{1 - s/2}{1 + s/2} = \frac{2 - s}{2 + s} \]
So, the characteristic equation becomes
\[ 1 - \frac{K(s - 2)}{s(s + 2)} = 0 \] \( \ldots (1) \)
Therefore, the open loop transfer function is
\[ G(s)H(s) = \frac{-K(s-2)}{s(s+2)} \]

Since, \( G(s)H(s) \) is negative, so the root locus will be complementary root locus and will exist at any point on the real axis, if the total number of poles and zeros to the right of that point is even.

So, the root locus (complementary) for the given system will exist on real axis in the region

\(-2 < s < 0 \) and \( s > 2 \)

The break points of the system are given by solution of

\[
\frac{dK}{ds} = 0
\]  

...(2)

From equation (1), we have

\[ K = \frac{s(s+2)}{(s-2)} \]

Substituting it in equation (2), we get

\[
\frac{d}{ds} \left[ \frac{s(s+2)}{(s-2)} \right] = 0
\]

or \( (s-2)(2s+2) - s(s+2) = 0 \)

or \( 2s^2 - 2s - 4 - s^2 - 2s = 0 \)

or \( s^2 - 4s - 4 = 0 \)

So,

\[ s = \frac{4 \pm 5.657}{2} = 4.83, -0.83 \]

Hence, break away point is \( s = -0.83 \) and break in point is \( s = 4.83 \).

**SOL 1.1.51**
Correct option is (B).

The sketch shows the variation of gain with respect to real axis, the maxima is found at \(-\sigma_1\) and minima is found at \(\sigma_2\).

Maxima indicates the breakaway point and minima indicates the break in point. Hence, \((-\sigma_1)\) is breakaway point and \(\sigma_2\) is break in point.

**SOL 1.1.52**
Correct option is (C).

For the given system, the characteristic equation is

\[ 1 + G(s)H(s) = 0 \]

or \[ 1 + \frac{K}{s(s+10)} = 0 \]

or \[ s^2 + 10s + 5 = 0 \]  \((K = 5)\)

So, the poles (roots) of system are

\[ s = \frac{-10 \pm 8.944}{2} \]

or \[ s = -9.47 \text{ and } s = -0.53 \]

**SOL 1.1.53**
Correct option is (B).

The actual change in the closed loop poles can be given by root sensitivity.
as converting the partial change to finite change, i.e.

\[ S'_k = \frac{K}{s} \frac{\Delta s}{\Delta K} \]

Hence, change in poles location is given as

\[ \Delta s = sS'_k \frac{\Delta K}{K} \]

Given that % change in \( K \) is 10. So, we have

\[ \frac{\Delta K}{K} \times 100 = 10 \]

or

\[ \frac{\Delta K}{K} = 0.1 \]  \( \quad \cdots(2) \)

Also, we have

\[ S'_k = -0.059 \text{ at } s = -9.47 \]

Substituting value of equation (2) and (3) in equation (1), we get

\[ \Delta s = (-9.47)(-0.059)(0.1) \]

\[ = 0.056 \]

Since, the change \( \Delta s \) is positive, so it moves in right side. Hence, the pole will move to the right by 0.056 units for a 10% change in \( K \).

**SOL 1.1.54**

Correct option is (D).

For the given system, the open loop poles and zeros are

- poles: \( s = 0 \) and \( s = 1 \)
- zero: \( s = -1 \)

So, we have the characteristic equation

\[ 1 + \frac{K(s+1)}{s(s-1)} = 0 \]

or

\[ K = -\frac{s(s-1)}{s+1} \]

The break points are given by solution of

\[ \frac{dK}{ds} = 0 \]

So,

\[ \frac{d}{ds} \left[ -\frac{s(s-1)}{s+1} \right] = 0 \]

or

\[ (s+1)(2s-1) - s(s-1) = 0 \]

or

\[ 2s^2 - s + 2s - 1 - s^2 + s = 0 \]

or

\[ s^2 + 2s - 1 = 0 \]

or

\[ s = \frac{-2 \pm 2.828}{2} \]

\[ = -1 \pm 1.414 \]  \( \cdots(1) \)

The root locus for the system is given below.
From the root locus, we get

\[ \text{centre of circle } = (-1,0) \]
and \[ \text{radius of circle } = 1.414 = \sqrt{2} \]

**SOL 1.1.55**
Correct option is (C).
For a system with open loop transfer function \( G(s)H(s) \), the criterion of root locus is

\[ \frac{G(s)H(s)}{s} = 180^\circ \]

or,

\[ \frac{G(s)H(s)}{s} = 180^\circ \]

Substituting \( s = \sigma + j\omega \) in above equation, we get

\[ \frac{G(s)H(s)}{s} = 180^\circ \]

or

\[ \tan^{-1}\left(\frac{\omega}{\sigma}\right) = 180^\circ + \tan^{-1}\left(\frac{\omega}{\sigma + 2}\right) \]  

or

\[ \tan\left[\tan^{-1}\left(\frac{\omega}{\sigma}\right) - \tan^{-1}\left(\frac{\omega}{\sigma + 2}\right)\right] = \tan\left[180^\circ + \tan^{-1}\left(\frac{\omega}{\sigma + 2}\right)\right] \]

or

\[ \frac{\omega}{\sigma + 3} - \frac{\omega}{\sigma} = 0 + \frac{\omega}{\sigma + 2} \]

or

\[ \frac{-3\omega}{\sigma + 3} + \omega = \frac{\omega}{\sigma + 2} \]

or

\[ -3(\sigma + 3) + \omega^2 = \sigma(\sigma + 3) + \omega^2 \]

or

\[ (\sigma^2 + 6\sigma + 9) + \omega^2 = -6 + 9 \]

or

\[ (\sigma + 3)^2 + \omega^2 = (\sqrt{3})^2 \]

This is the equation of circle.

**SOL 1.1.56**
Correct option is (D).
Given open loop transfer function of the system,

\[ G(s)H(s) = \frac{K}{(s + 1 + j)(s + 1 - j)(s + 3 + j)(s + 3 - j)} \]

The root locus starts from

\[ s_1 = -1 + j \]
\[ s_2 = -1 - j \]
\[ s_3 = -3 - j \]
\[ s_4 = -3 + j \]

Since, there is no zero, all root loci end at infinity. So, we have

Number of open loop poles, \( P = 4 \)
Number of open loop zeros, \( Z = 0 \)

Therefore, number of asymptotes is 4 with angles of

\[ \phi_A = 45^\circ, 135^\circ, 225^\circ, 315^\circ \]

Also, the point of intersection (centroid) of asymptotes with real axis is

\[ \sigma_A = \frac{\Sigma \text{Re}[P] - \Sigma \text{Re}[Z]}{P - Z} \]

\[ = \frac{(-1) + (-1) + (-3) + (-3) - 0}{4 - 0} = -2 \]

So, we get the root locus of the system as shown below.
Thus, there is no breakaway point.

Correct option is (D).

The characteristic equation of the system is

\[ 1 + G(s)H(s) = 0 \]

or

\[ 1 + \frac{K}{s(s+4)(s+5)} = 0 \]

or

\[ s^3 + 9s^2 + 20s + K = 0 \]

...(1)

Intersection of root loci with \( j\omega \) axis is determined using Routh’s array. For the given system, we form the Routh’s array as

| \( s^3 \) | 1 | 20 |
| \( s^2 \) | 9 | \( K \) |
| \( s^1 \) | \( 180 - K \) \( \frac{y}{y} \) |
| \( s^0 \) | \( K \) |

The critical gain before the closed loop system goes to instability is \( K_c = 180 \) and the auxiliary equation is

\[ 9s^2 + 180 = 0 \]

or

\[ s^2 = -20 \]

or

\[ s = \pm j2\sqrt{5} \]

Hence, root loci intersect with \( j\omega \)-axis at \( s = \pm j2\sqrt{5} \). The gain margin for \( K = 18 \) is given by,

\[ GM \ (\text{in dB}) = 20\log_{10}\left(\frac{K}{K_c}\right) \]

\[ = 20\log_{10}\left(\frac{180}{18}\right) = 20 \text{ dB} \]

The gain margin for \( K = 1800 \) is given by

\[ GM \ (\text{in dB}) = 20\log_{10}\left(\frac{180}{1800}\right) = -20 \text{ dB} \]

The break away point is given by solution of

\[ \frac{dK}{ds} = 0 \]

From equation (1), we have

\[ K = -(s^3 + 9s^2 + 20s) \]

So,

\[ \frac{dK}{ds} = -(3s^2 + 18s + 20) = 0 \]

or

\[ s = -\frac{18 \pm 9.165}{6} \]
Point \( s = -1.4725 \) lies on root locus. So, break away point is \( s = -1.4725 \).

The value of \( K \) at break away point is

\[
K = \left| s(s + 4)(s + 5) \right| = 13.128 \quad (s = -1.4725)
\]

Correct option is (D).

The characteristic equation of the system is

\[
1 + \frac{K(s + 1)}{s(s - 1)(s^2 + 4s + 16)} = 0 \quad ...(1)
\]

or \[ s^4 + 3s^3 + 12s^2 + (K - 16)s + K = 0 \]

Intersection of root loci with \( j\omega \)-axis is determined using Routh’s array which is shown below.

<table>
<thead>
<tr>
<th>( s^4 )</th>
<th>1</th>
<th>12</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^3 )</td>
<td>3</td>
<td>( K - 16 )</td>
<td></td>
</tr>
<tr>
<td>( s^2 )</td>
<td>( \frac{52 - K}{3} )</td>
<td>( K )</td>
<td></td>
</tr>
<tr>
<td>( s^1 )</td>
<td>( -\frac{K^2 + 59K - 832}{52 - K} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s^0 )</td>
<td>( K )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The root locus cross the \( j\omega \)-axis, if \( s^1 \) row is completely zero, i.e.

\[-\frac{K^2 + 59K - 832}{52 - K} = 0\]

or \[ K^2 - 59K + 832 = 0 \]

or \[ K = \frac{59 \pm 12.37}{2} = 35.7, 23.3 \]

Hence, root locus cross \( j\omega \)-axis two times and the break points are given by solution of

\[
\frac{dK}{ds} = 0
\]

Also, we can directly check option using pole-zero plot.

So, break away point will lie on real axis from \( s = 0 \) to 1 and break in point will lie on real axis for \( s < -1 \). Hence,

\[ s = 0.45 \] is break away point

and \[ s = -2.26 \] is break in point

************
SOL 1.2.1

Correct answer is $-1.45$.

Given root locus of $u/fb$ system is

Here, root locus branches meet between $-1$ and $-2$ and go apart. Hence, break-away point will lie between $-1$ and $-2$. For this system, the open loop poles and zeros are:

- Zeros: $s = 3$ and $s = 5$
- Poles: $s = -1$ and $s = -2$

So, the transfer function of given system will be

$$G(s) = \frac{K(s-3)(s-5)}{(s+1)(s+2)}$$

Therefore, the characteristic equation is obtained as

$$1 + G(s)H(s) = 0$$

or

$$1 + \frac{K(s-3)(s-5)}{(s+1)(s+2)} = 0$$

or

$$K = \frac{-(s^2 + 3s + 2)}{(s^2 - 8s + 15)} ... (1)$$

Differentiating equation (1) with respect to $s$ and equating to zero, we have

$$\frac{dK}{ds} = \frac{-(s^2 - 8s + 15)(2s + 3) + (s^2 + 3s + 2)(2s - 8)}{(s^2 - 8s + 15)^2} = 0$$

or

$$11s^2 - 26s - 61 = 0$$

or

$$s = +3.9$$ and $$s = -1.45$$

Thus, $s = -1.45$ is break-away point and $s = +3.9$ is break-in point.

SOL 1.2.2

Correct answer is $8.62$.

Break-away and break-in points always satisfy characteristic equation. So, we substitute $s = -1.45$ in equation (1) to obtain

$$K = \frac{-[(-1.45)^2 + 3(-1.45) + 2]}{[(-1.45)^2 - 8(-1.45) + 15]}$$

$$= \frac{(-0.2475)}{-28.7025}$$

$$= 8.62 \times 10^{-3}$$
SOL 1.2.3
Correct answer is 108.4.
Forward path transfer function of given uf system is
\[ G(s) = \frac{K(s+2)}{(s+3)(s^2 + 2s + 2)} \]
So, we have the open loop poles and zeros as
zero: \( s = -2 \)
poles: \( s = -3 \) and \( s = -1 \pm j1 \)
Therefore, we get the pole-zero plot as

![Pole-Zero Plot](image)

Angle of departure at pole \( P_1 \) is given by
\[ \phi_D = \pm [180^\circ + \phi] \]
where \( \phi \) is net angle contribution at pole \( P_1 \) due to all other poles and zeros.
\[ \phi = \phi_2 - \phi_P \]
\[ = \phi_2 - [\phi_2 + \phi_3] \]
where \( \phi_2 = \tan^{-1}1; \phi_2 = 90^\circ; \phi_3 = \tan^{-1}\frac{1}{2} \)
So,
\[ \phi = \tan^{-1}1 - [90^\circ + \tan^{-1}\frac{1}{2}] \]
Therefore, we obtain the departure angle as
\[ \phi_D = \pm 108.4^\circ \]
Hence, departure angle for pole \( P_1 \) is \(+108.4^\circ\) and departure angle for pole \( P_2 \) is \(-108.4^\circ\) because \( P_1 \) and \( P_2 \) are complex conjugate.

SOL 1.2.4
Correct answer is 3.162.
Given root locus is shown below.

![Root Locus Plot](image)

It does not have any zero and have poles at \( s = -4 \) and \( s = -1 \pm j1 \). So, the
The open loop transfer function is

\[ G(s) = \frac{K}{(s + 4)(s^2 + 2s + 2)} \]

\[ = \frac{K}{s^3 + 6s^2 + 10s + 8} \]

Therefore, we get the closed loop transfer function as

\[ T(s) = \frac{K}{s^3 + 6s^2 + 10s + 8} \]

The characteristic equation of the system is given as

\[ s^3 + 6s^2 + 10s + 8 + K = 0 \]

So, we have the Routh’s array as

<table>
<thead>
<tr>
<th>s^3</th>
<th>1</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>s^2</td>
<td>6</td>
<td>8 + K</td>
</tr>
<tr>
<td>s^1</td>
<td>52 - K</td>
<td>6</td>
</tr>
<tr>
<td>s^0</td>
<td>8 + K</td>
<td></td>
</tr>
</tbody>
</table>

Root locus will cut imaginary axis, if element in \( s^1 \) is zero, i.e.

\[ \frac{52 - K}{6} = 0 \Rightarrow K = 52 \]

So, we have the auxiliary equation

\[ 6s^2 + (8 + 52) = 0 \]

or

\[ s^2 = -\frac{60}{6} = -10 \]

or

\[ s = \pm 3.162 \]

**SOL 1.2.5**

Correct answer is 17.

For the given system, we have the pole-zero plot as shown below.

Gain \( K \) at any \( s = s_0 \) point on root locus is given by

\[ K_{s=s_0} = \frac{\text{Product of phasors drawn from OLP at that point}}{\text{Product of phasors drawn from OLZ at that point}} \]

Since, no any zero is present in the given system. So, we obtain

\[ K = \text{Product of phasors drawn from OLP at that point} \]

So,

\[ K_{s=-5} = (PP_3) \times (PP_1) \times (PP_2) \]

\[ = 1 \times \sqrt{4^2+1} \times \sqrt{4^2+1} \]

\[ = 4^2 + 1 = 17 = 17 \]
SOL 1.2.6 Correct answer is 600.
Open loop transfer function of given system is
\[ G(s)H(s) = \frac{K(s+8)}{s(s+4)(s+12)(s+20)} \]
For the given system, we have the open loop poles and zeros as
Zeros : \( Z = -8 \)
Poles : \( P_1 = 0; P_2 = -4; P_3 = -12; P_4 = -20 \)
So, we get the pole-zero plot for the given system as

Therefore, we obtain the value of \( K \) at \( s = -10 \) as
\[ K \bigg|_{s = -10} = \frac{\pi (\text{Phasors drawn from OLP at } s = -10)}{\pi (\text{Phasors drawn from OLZ at } s = -10)} \times \frac{(PP_1) \times (PP_2) \times (PP_3) \times (PP_4)}{(PZ)} = \frac{10 \times 6 \times 2 \times 10}{2} = 600 \]

SOL 1.2.7 Correct answer is \(-1\).
Characteristic equation of given closed loop system is
\[ s(s+1)(s+2) + K = 0 \]
or
\[ 1 + \frac{K}{s(s+1)(s+2)} = 0 \]
Since, the characteristic equation for a system is defined as
\[ 1 + G(s)H(s) = 0 \]
So, we get open loop transfer function of the system as
\[ G(s)H(s) = \frac{K}{s(s+1)(s+2)} \]
Therefore, we have
Poles: \( s = 0, s = -1, s = -2 \)
Zeros: No zero
Thus, the centroid is obtained as
\[ \sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P - Z} = \frac{(0 - 1 - 2) - (0)}{3 - 0} = -\frac{3}{3} = -1 \]

SOL 1.2.8 Correct answer is 10.
Given the open loop transfer function,
\[ G(s) = \frac{K}{s(s^2 + 7s + 12)}; H(s) = 1 \]
So, we have the characteristic equation
\[ 1 + G(s)H(s) = 0 \]
or \[1 + \frac{K}{s(s^2 + 7s + 12)} = 0\]

or \[s^3 + 7s^2 + 12s + K = 0\] ...(1)

If point \(s = -1 + j1\) lies on root locus, then it satisfies characteristic equation.

Substituting \(s = -1 + j1\) in equation (1), we get

\[(-1 + j)^3 + 7(-1 + j)^2 + 12(-1 + j) + K = 0\]

or \[-10 + K = 0\]

So, \(K = 10\)

**SOL 1.2.9**

Correct answer is 71.56.

Given the open loop transfer function,

\[G(s)H(s) = \frac{K(s^2 + 2s + 10)}{(s^2 + 6s + 10)}\]

So, we have the open loop poles and zeros as

Poles: \(s^2 + 6s + 10 = 0 \Rightarrow s = -3 \pm j1\)

zeros \(s^2 + 2s + 10 = 0 \Rightarrow s = -1 \pm j3\)

Therefore, we get the pole-zero plot in \(s\)-plane as

Angle of departure at complex pole is given by

\[\phi_D = \pm [180^\circ + \phi]\]

where \(\phi\) is the net angle contribution at this pole due to all other poles and zeros. So, we have

\[\phi = \phi_{z1} + \phi_{z2} - \phi_P\]

where

\[\phi_{z1} = -(90^\circ + \theta) = -\left(90^\circ + \tan^{-1} \frac{2}{7}\right) = -135^\circ;\]

\[\phi_{z2} = 180^\circ - \tan^{-1} \frac{4}{3} = 116.56^\circ;\]

and \(\phi_P = 90^\circ\)

Therefore, we get

\[\phi = -135^\circ + 116.56^\circ - 90^\circ = -108.44^\circ\]

Hence, angle of departure will be

\[\phi_D = \pm [180^\circ - 108.44^\circ] = \pm [71.56^\circ]\]
SOL 1.2.10  Correct answer is 198.43.
The pole zero plot of the system is

The angle of arrival at complex zero is given by
\[ \phi_A = \pm [180^\circ - \phi] \]
where, \( \phi \) is the net angle contribution at this zero due to all other poles and zeros. So, we have
\[ \phi = \phi_Z - \phi_P - \phi_P \]
where
\[ \phi_Z = 90^\circ \]
\[ \phi_P = \tan^{-1}\frac{2}{2} = 45^\circ \]
\[ \phi_P = \tan^{-1}\frac{4}{2} = 63.43^\circ \]
Hence,
\[ \phi = 90^\circ - 45^\circ - 63.43^\circ \]
\[ = -18.43^\circ \]
Thus, we obtain the angle of arrival as
\[ \phi_A = \pm [180^\circ - \phi] \]
\[ = \pm [180^\circ + 18.43^\circ] \]
\[ = \pm [198.43^\circ] \]

SOL 1.2.11  Correct answer is 5.2.
The poles of the open loop transfer function are
\[ s = -2 \] and \[ s = -10 \]
So, the root loci starts at \( s = -2 \) and \( s = -10 \). Also, we have
Number of poles, \( P = 2 \)
Number of zeros, \( Z = 0 \)
Hence, number of asymptotes is obtained as
\[ P - Z = 2 \]
Therefore, the angle of asymptotes is given by
\[ \phi_a = \frac{(2q+1)180^\circ}{P-Z}; \quad q = 0, 1, 2, \ldots, (P-Z-1) \]
So,
\[ \phi_a = 90^\circ; \quad q = 0 \]
and
\[ \phi_a = 270^\circ; \quad q = 1 \]
Again, the open loop transfer function is
\[ G(s)H(s) = \frac{10K}{(s+2)(s+10)} \]
So,
\[ |G(s)H(s)| \leq \frac{10K}{(s+2)(s+10)} = 1 \]
The break away point is given by solution of
\[
\frac{dK}{ds} = 0
\]
Substituting equation (1) in the above expression, we have
\[
(s + 2) + (s + 10) = 0
\]
or
\[
2s + 12 = 0
\]
or
\[
s = -6
\]
The point \(s = (-6, j0)\) is the break-away point. Therefore, root locus will be as shown below.

\[
\xi = \frac{1}{\sqrt{2}}
\]
and
\[
\theta = 45^\circ
\]
Root locus will cross constant \(\xi = \frac{1}{\sqrt{2}}\) line at point \(p\). The intersection point \(p\) is \((-6, j\omega)\). At the point \(p\), angle of function is
\[
\theta_i = 135^\circ
\]
which is given by
\[
135^\circ = \tan^{-1}\left(\frac{\omega}{6}\right)
\]
or
\[
135^\circ = 180^\circ - \tan^{-1}\left(\frac{\omega}{6}\right)
\]
or
\[
\tan^{-1}45^\circ = \frac{\omega}{6}
\]
\[
\omega = 6
\]
Hence, point \(p\) will be \((-6 + j6)\). So, the gain \(K\) at point \(p\) is
\[
K = \left|\frac{s + 2}{s + 10}\right|_{s=-6+j6} = \left|\frac{-4 + j6}{10}\right| = 5.2
\]

**SOL 1.2.12**
Correct answer is 12.04.
From root locus, intersection with imaginary axis indicates the marginal stability. So, for marginal stability the value of \(K = 48\). So, the gain margin is given by
\[
\text{Gain margin } (GM) = \frac{\text{Value of } K \text{ for marginal stability}}{\text{desired value of } K} = \frac{48}{12} = 4
\]
In decibel, we get
\[
GM \text{ (in dB)} = 20\log_{10}4 = 12.04 \text{ dB}
\]
SOL 1.2.13
Correct answer is 385.
For the given system, we have
\[1 + G(s)H(s) = 0\]
or
\[|G(s)H(s)| = \left| \frac{K}{s(s + 10)(s + 20)} \right| = 1\]
or
\[K = s(s + 10)(s + 20)\]
The break points of the system are given by solution of
\[\frac{dK}{ds} = 0\]
or
\[\frac{ds}{ds^3 + 30s^2 + 200s} = 0\]
or
\[3s^2 + 60s + 200 = 0\]
So,
\[s = -60 \pm 34.64\]
\[= -15.773, -4.226\]
Therefore, we get the pole zero plot as

\[\text{Hence, break away point is}\]
\[s = -4.226\]
The value of \(K\) at point \(s_0\) is given by
\[|G(s)H(s)| = 1 \text{ at } s = s_0\]
or
\[\left. \left| \frac{K}{s(s + 10)(s + 20)} \right| \right|_{s=s_0} = 1\]
So,
\[K = \left. \left| \frac{K}{s} \right| \right|_{s=s_0}\]
where \(s_0 = -4.226\) is break away point, given as
\[K = 4.226 \times 5.774 \times 15.774\]
\[= 384.9 \approx 385\]

SOL 1.2.14
Correct answer is -0.059.
The root sensitivity is defined as the ratio of the fractional change in a closed loop pole to the fraction change in a system parameters, such as gain \(K\). We calculate the sensitivity of a closed loop pole, \(s\), to gain \(K\),
\[S_K = \frac{\delta s}{s\delta K} = \frac{K\delta s}{s\delta K} \quad \ldots(1)\]
where \(s\) is the current pole location and \(K\) is the current gain. The characteristic equation for the system is
\[s^3 + 10s + K = 0 \quad \ldots(2)\]
Differentiating equation (2) with respect to \(K\), we have
\[2s\frac{\delta s}{\delta K} + 10\frac{\delta s}{\delta K} + 1 = 0\]
or
\[\frac{\delta s}{\delta K} = \frac{-1}{2s + 10}\]
From equation (1), the root sensitivity is obtained as
\[
S_k^* = \frac{K}{s^2 + 10} - \frac{1}{2s + 10}
\]
\[
= -\frac{K}{s(2s + 10)}
\]
The root sensitivity at \( s = -9.47 \) is given by substituting \( s = -9.47 \) and corresponding \( K = 5 \) in above expression, we get
\[
S_k^* = -\frac{5}{(-9.47)(-18.94 + 10)}
\]
\[
s_k^* = -0.059
\]

\textbf{SOL 1.2.15}
Correct answer is 0.2.

For the given system, we have the characteristic equation
\[
1 + G(s)H(s) = 0
\]
or
\[
1 + \frac{K(s^2 + 4)}{s(s + 2)} = 0
\]
or
\[
K = -\frac{(s^2 + 2s)}{s^2 + 4}
\]
The break points of the root locus are given by solution of
\[
\frac{dK}{ds} = 0
\]
or
\[
-\left[(s^2 + 4)(2s + 2) - (s^2 + 2s)(2s)\right] = 0
\]
or
\[
2s^2 - 8s - 8 = 0
\]
or
\[
s^2 - 4s - 4 = 0
\]
So,
\[
s = 4.82, -0.82
\]
The root locus will be on real axis at any point, if total number of poles and zeros are odd to the right of that point.

Hence, breakaway point should lie between \( s = 0 \) and \(-2\). So, breakaway point is \( s = -0.82 \). Now, the open loop transfer function is
\[
G(s)H(s) = \frac{K(s^2 + 4)}{s(s + 2)}
\]
So, we obtain the value of \( K \) at break-away point as
\[
\left|G(s)H(s)\right| = 1 \text{ at } s = -0.82
\]
or
\[
\frac{K\left(-0.82\right)^2 + 4}{\left(-0.82\right)\left(-0.82 + 2\right)} = 1
\]
or
\[
K = \frac{0.82 \times 1.18}{4.67} = 0.2
\]

\textbf{SOL 1.2.16}
Correct answer is 0.47.

From root locus, for \( K = 12.2 \) poles are complex conjugate and are given by \( s = -0.7 \pm j1.3 \).
For $K = 12.2$, the pole zero plot is

From the pole-zero plot, we have

$$\theta = \tan^{-1}\left(\frac{1.3}{0.7}\right) = 61.69$$

So, the damping ratio is given by

$$\xi = \cos \theta$$

$$= \cos(61.69)$$

$$= 0.47$$

**SOL 1.2.17**

Correct answer is 0.187.

The peak overshoot ($M_p$) is defined as

$$M_p = e^{-\xi \sqrt{1-\xi^2}}$$

For $K = 12.2$, damping ratio is $\xi = 0.47$. So, we have

$$M_p = e^{-0.47 \sqrt{1-0.47^2}}$$

$$= 0.187$$

**SOL 1.2.18**

Correct answer is $-1$.

The overall transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{1}{s(s+1)} \frac{1}{1 + \frac{1}{s(s+1)(\alpha s + 1)}}$$

$$= \frac{1}{s^2 + (\alpha + 1)s + 1}$$

So, we have the characteristic equation as

$$s^2 + (\alpha + 1)s + 1 = 0$$

or

$$s^2 + s + 1 + \alpha s = 0$$

or

$$1 + \frac{\alpha s}{s^2 + s + 1} = 0$$

Comparing it to $1 + G(s)H(s) = 0$, we have the open loop transfer function

$$G(s)H(s) = \frac{\alpha s}{s^2 + s + 1}$$

Also, we have

$$\alpha = \frac{-(s^2 + s + 1)}{s}$$

The break points are given by solution of

$$\frac{d\alpha}{ds} = 0$$
So, \[-\frac{s(2s + 1) - (s^2 + s + 1)}{s} = 0\]
or \[s^2 - 1 = 0\]
or \[s = \pm 1\]
The point \(s = 1\) does not lie on root locus and \(s = -1\) lies on root locus. Hence, break away point is \(s = -1\).

**SOL 1.2.19**

Correct answer is 1.

We know that, on the break away point system has multiple poles. If break away point is on real axis, then multiple poles should be real and equal, and in this case, system have critically damped response. Hence, value of parameter \(\alpha\) for critical damping will be equal to value of \(\alpha\) at break away point. For the given system, breakaway point is \(s = -1\).

So, the value of \(\alpha\) at \(s = -1\) is obtained as

\[\left|\frac{G(s)H(s)}{s}\right| = 1\]

or

\[\left|\frac{\alpha}{s^2 + s + 1}\right|_{s=-1} = 1\]

\[\alpha = \frac{|(-1)^2 + (-1) + 1|}{|(-1)|} = 1\]

**SOL 1.2.20**

Correct answer is 27.

The root locus plot gives the location of the closed loop poles for different values of parameter gain \(K\). So, we have the characteristic equation as

\[1 + \frac{K(s + 1)}{s^2(s + 9)} = 0\]

or

\[s^3 + 9s^2 + Ks + K = 0\]  \hspace{1cm} ...(1)

For all the roots to be equal and real, we require

\[(s + P)^3 = s^3 + 3Ps^2 + 3P^2s + P^3 = 0\]  \hspace{1cm} ...(2)

On comparing equations (1) and (2), we get

\[3P = 9 \Rightarrow P = 3\]

and

\[K = P^3\]

\[= (3)^3\]

\[= 27\]

**SOL 1.2.21**

Correct answer is 29.

First we check if point lies on root locus. For this, we use angle criterion

\[\frac{G(s)H(s)}{s} = \pm 180\]

Since, we have

\[G(s)H(s) \mid_{s = -3 + j\beta} = \frac{K}{(-3 + j\beta)(-3 + j\beta + 5)}\]

\[= \frac{K}{(-2 + j\beta)(2 + j\beta)}\]

So,

\[\frac{G(s)H(s)}{s} \mid_{s = -3 + j\beta} = -\tan^{-1}(\frac{3}{2}) - \tan^{-1}(\frac{5}{2})\]

\[= -180^0 + \tan^{-1}\frac{5}{2} - \tan^{-1}\frac{5}{2}\]

\[= -180^0\]
i.e. the given point satisfies angle criterion. Now, using magnitude condition, we have

\[ |G(s)H(s)|_{\arg=-3+\jmath 5} = 1 \]

or

\[ \frac{K}{(-2+j5)(2+j5)} = 1 \]

or

\[ \frac{K}{\sqrt{4+25}\sqrt{4+25}} = 1 \]

Thus,

\[ K = 29 \]

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SOLUTIONS 1.3

SOL 1.3.1 Correct option is (D).

SOL 1.3.2 Correct option is (B).

SOL 1.3.3 Correct option is (C).

SOL 1.3.4 Correct option is (C).

SOL 1.3.5 Correct option is (B).

SOL 1.3.6 Correct option is (A).

SOL 1.3.7 Correct option is (A).

SOL 1.3.8 Correct option is (C).

SOL 1.3.9 Correct option is (A).

SOL 1.3.10 Correct option is (A).

SOL 1.3.11 Correct option is (B).

SOL 1.3.12 Correct option is (A).
The number of branches is equal to the order of the polynomial. Here, order of the system is 4. Hence the number of branches is 4.

SOL 1.3.13 Correct option is (C).
The number of asymptotes = # open loop poles − # open loop zeros

= 5 − 3 = 2

SOL 1.3.14 Correct option is (B).

SOL 1.3.15 Correct option is (C).

SOL 1.3.16 Correct option is (C).

SOL 1.3.17 Correct option is (C).
SOL 1.3.18 Correct option is (B).
The open-loop poles are, 
\[ s = -2 \pm j2, s = -1 \] and \( s = 0 \) and the open loop zero is \( s = -3 \)
The angle of asymptotes is given by 
\[ \phi_A = \frac{(2q + 1)180^\circ}{P - Z}; q = 0, 1, \ldots, (P - Z - 1) \]
So, \( \phi_A = 60^\circ; q = 0 \)
and \( \phi_A = 180^\circ; q = 1 \)
and \( \phi_A = 300^\circ - 60^\circ; q = 2 \)
The centroid is 
\[ \sigma_A = \frac{\Sigma \text{real of poles} - \Sigma \text{real of zeros}}{P - Z} \]
\[ = \frac{\{-2\} + \{-2\} + \{-1\} + 0} - (-3) \]
\[ = -\frac{2}{3} \]

SOL 1.3.19 Correct option is (A).
When the system has real and different poles then response becomes non oscillatory. From root locus plot, it can be observed that for \( 0 < K < 0.4 \) system has real and different poles.

SOL 1.3.20 Correct option is (D).
Due to addition of zero to the open loop transfer function, root locus move to left half. And due to addition of pole to the open loop transfer function, root locus move to right half.

SOL 1.3.21 Correct option is (D).
The meeting point of asymptotes on the real axis is a centroid which is given by 
\[ \sigma_A = \frac{\Sigma \text{real of poles} - \Sigma \text{real of zeros}}{\# \text{poles} - \# \text{zeros}} \]
\[ = \frac{\{0 + \{-2\} + \{-4\} + \{-1\} + \{-1\}\} - (-5)}{5 - 1} \]
\[ = -\frac{3}{4} = -0.75 \]

SOL 1.3.22 Correct option is (A).
From the root locus we can observe that the four open loop poles lie as \( s = -1 \). So, open loop transfer function is, 
\[ G(s) = \frac{K}{(s + 1)^4} \]
The characteristic equation is 
\[ 1 + \frac{K}{(s + 1)^4} = 0 \]
or \[ s^4 + 4s^3 + 6s^2 + 4s + 1 + K = 0 \]
The Routh’s array is
For intersection of $j\omega$-axis, $s^4$ row should be completely zero.

i.e. \[ \frac{16 - 4K}{5} = 0 \]

or \[ K = 4 \]

SOL 1.3.23
Correct option is (B).

The intersection of asymptotes is always on the real axis because it is given by

\[ \sigma_A = \frac{\Sigma \text{real part of } OLP - \Sigma \text{real part of } OLZ}{\# OLP - \# OLZ} \]

The breakaway point is determined by solution of \[ \frac{dK}{ds} = 0 \].

So, it can be real or complex.

SOL 1.3.24
Correct option is (A).

The effect of compensating pole is to pull the root locus towards right half of $s$-plane. The effect of compensating zero is to pull the root locus towards left half of $s$-plane.

SOL 1.3.25
Correct option is (A).

1. There will be four asymptotes because,
\[ \# OLP - \# OLZ = 4 - 0 = 4 \]
2. There will be four separate root because the order of polynomial is four.
3. Asymptotes will intersect at $\sigma_A$,
\[ \sigma_A = \frac{(0 - 1 - 2 - 3) - (0)}{4 - 0} = -\frac{6}{4} = -\frac{3}{2} \]

SOL 1.3.26
Correct option is (C).

The given characteristic equation is
\[ 1 + \frac{K}{s(s + 1)(s + 2)} = 0 \]

or \[ 1 + G(s)H(s) = 0 \]

So, the open loop transfer function is
\[ G(s)H(s) = \frac{K}{s(s + 1)(s + 2)} \]

The centroid $\sigma_A$ is,
\[ \sigma_A = \frac{(0 - 1 - 2) - (0)}{3 - 0} = -1 \]

SOL 1.3.27
Correct option is (A).

SOL 1.3.28
Correct option is (C).
The value of gain $K$ at any point $s_0$ on the root locus is given by

$$K \bigg|_{s=s_0} = \frac{\Pi(\text{Lengths of vectors from } OLP \text{ to the point } s_0)}{\Pi(\text{Lengths of vectors from } OLZ \text{ to the point } s_0)}$$

**SOL 1.3.29**
Correct option is (A).
The root locus is symmetrical about real axis, not $j\omega$-axis.

**SOL 1.3.30**
Correct option is (A).

**SOL 1.3.31**
Correct option is (C).

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