To Our Parents
Preface to the Series

For almost a decade, we have been receiving tremendous responses from GATE aspirants for our earlier books: GATE Multiple Choice Questions, GATE Guide, and the GATE Cloud series. Our first book, GATE Multiple Choice Questions (MCQ), was a compilation of objective questions and solutions for all subjects of GATE Electronics & Communication Engineering in one book. The idea behind the book was that Gate aspirants who had just completed or about to finish their last semester to achieve his or her B.E/B.Tech need only to practice answering questions to crack GATE. The solutions in the book were presented in such a manner that a student needs to know fundamental concepts to understand them. We assumed that students have learned enough of the fundamentals by his or her graduation. The book was a great success, but still there were a large ratio of aspirants who needed more preparatory materials beyond just problems and solutions. This large ratio mainly included average students.

Later, we perceived that many aspirants couldn’t develop a good problem solving approach in their B.E/B.Tech. Some of them lacked the fundamentals of a subject and had difficulty understanding simple solutions. Now, we have an idea to enhance our content and present two separate books for each subject: one for theory, which contains brief theory, problem solving methods, fundamental concepts, and points-to-remember. The second book is about problems, including a vast collection of problems with descriptive and step-by-step solutions that can be understood by an average student. This was the origin of GATE Guide (the theory book) and GATE Cloud (the problem bank) series: two books for each subject. GATE Guide and GATE Cloud were published in three subjects only.

Thereafter we received an immense number of emails from our readers looking for a complete study package for all subjects and a book that combines both GATE Guide and GATE Cloud. This encouraged us to present GATE Study Package (a set of 10 books: one for each subject) for GATE Electronic and Communication Engineering. Each book in this package is adequate for the purpose of qualifying GATE for an average student. Each book contains brief theory, fundamental concepts, problem solving methodology, summary of formulae, and a solved question bank. The question bank has three exercises for each chapter: 1) Theoretical MCQs, 2) Numerical MCQs, and 3) Numerical Type Questions (based on the new GATE pattern). Solutions are presented in a descriptive and step-by-step manner, which are easy to understand for all aspirants.

We believe that each book of GATE Study Package helps a student learn fundamental concepts and develop problem solving skills for a subject, which are key essentials to crack GATE. Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge all constructive comments, criticisms, and suggestions from the users of this book. You may write to us at rajkumar.kanodia@gmail.com and ashish.murolia@gmail.com.

Acknowledgements

We would like to express our sincere thanks to all the co-authors, editors, and reviewers for their efforts in making this project successful. We would also like to thank Team NODIA for providing professional support for this project through all phases of its development. At last, we express our gratitude to God and our Family for providing moral support and motivation.

We wish you good luck!
R. K. Kanodia
Ashish Murolia
Engineering Mathematics (EC, EE, and IN Branch)

**Linear Algebra:** Matrix Algebra, Systems of linear equations, Eigen values and eigen vectors.

**Calculus:** Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, Partial Derivatives, Maxima and minima, Multiple integrals, Fourier series. Vector identities, Directional derivatives, Line, Surface and Volume integrals, Stokes, Gauss and Green’s theorems.

**Differential equations:** First order equation (linear and nonlinear), Higher order linear differential equations with constant coefficients, Method of variation of parameters, Cauchy’s and Euler’s equations, Initial and boundary value problems, Partial Differential Equations and variable separable method.

**Complex variables:** Analytic functions, Cauchy’s integral theorem and integral formula, Taylor’s and Laurent’ series, Residue theorem, solution integrals.

**Probability and Statistics:** Sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Discrete and continuous distributions, Poisson, Normal and Binomial distribution, Correlation and regression analysis.

**Numerical Methods:** Solutions of non-linear algebraic equations, single and multi-step methods for differential equations.

**Transform Theory:** Fourier transform, Laplace transform, Z-transform.

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Engineering Mathematics (ME, CE and PI Branch)

**Linear Algebra:** Matrix algebra, Systems of linear equations, Eigen values and eigen vectors. Calculus: Functions of single variable, Limit, continuity and differentiability, Mean value theorems, Evaluation of definite and improper integrals, Partial derivatives, Total derivative, Maxima and minima, Gradient, Divergence and Curl, Vector identities, Directional derivatives, Line, Surface and Volume integrals, Stokes, Gauss and Green’s theorems.

**Differential equations:** First order equations (linear and nonlinear), Higher order linear differential equations with constant coefficients, Cauchy’s and Euler’s equations, Initial and boundary value problems, Laplace transforms, Solutions of one dimensional heat and wave equations and Laplace equation.

**Complex variables:** Analytic functions, Cauchy’s integral theorem, Taylor and Laurent series.

**Probability and Statistics:** Definitions of probability and sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Poisson, Normal and Binomial distributions.

**Numerical Methods:** Numerical solutions of linear and non-linear algebraic equations Integration by trapezoidal and Simpson’s rule, single and multi-step methods for differential equations.

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CHAPTER 1

MATRIX ALGEBRA

1.1 INTRODUCTION

This chapter, concerned with the matrix algebra, includes the following topics:
• Multiplication of matrix
• Transpose of matrix
• Determinant of matrix
• Rank of matrix
• Adjoint of matrix
• Inverse of matrix: elementary transformation, determination of inverse using elementary transformation
• Echelon form and normal form of matrix; procedure for reduction of normal form.

1.2 MULTIPLICATION OF MATRICES

If \( A \) and \( B \) be any two matrices, then their product \( AB \) will be defined only when number of columns in \( A \) is equal to the number of rows in \( B \). If

\[
A = [a_{ij}]_{m \times n}
\]

and

\[
B = [b_{jk}]_{n \times p}
\]

then their product,

\[
AB = C = [c_{ik}]_{m \times p}
\]

will be matrix of order \( m \times p \), where

\[
c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}
\]

PROPERTIES OF MATRIX MULTIPLICATION

If \( A, B \) and \( C \) are three matrices such that their product is defined, then
1. Generally not commutative; \( AB \neq BA \)
2. Associative law; \( (AB)C = A(BC) \)
3. Distributive law; \( A(B + C) = AB + AC \)
4. Cancellation law is not applicable, i.e. if \( AB = AC \), it does not mean that \( B = C \).
5. If \( AB = 0 \), it does not mean that \( A = 0 \) or \( B = 0 \).
6. \( (AB)^T = (BA)^T \)
1.3 TRANSPOSE OF A MATRIX

The matrix obtained form a given matrix $A$ by changing its rows into
columns or columns int rows is called Transpose of matrix $A$ and is denoted
by $A^T$. From the definition it is obvious that if order of $A$ is $m \times n$, then
order of $A^T$ is $n \times m$.

**PROPERTIES OF TRANSPOSE OF MATRIX**

Consider the two matrices $A$ and $B$

1. $(A^T)^T = A$
2. $(A \pm B)^T = A^T \pm B^T$
3. $(AB)^T = B^T A^T$
4. $(kA)^T = k(A^T)$
5. $(A_1A_2A_3...A_{n-1}A_n)^T = A_n^T A_{n-1}^T ... A_2^T A_1^T$

1.4 DETERMINANT OF A MATRIX

The determinant of square matrix $A$ is defined as

$$|A| = \left| \begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\
\end{array} \right|$$

**PROPERTIES OF DETERMINANT OF MATRIX**

Consider the two matrices $A$ and $B$.

1. $|A|$ exists $\iff A$ is a square matrix
2. $|AB| = |A| |B|$
3. $|A^T| = |A|$
4. $|kA| = k^n |A|$, if $A$ is a square matrix of order $n$.
5. If $A$ and $B$ are square matrices of same order then $|AB| = |BA|$.
6. If $A$ is a skew symmetric matrix of odd order then $|A| = 0$.
7. If $A = \text{diag} (a_1, a_2, ..., a_n)$ then $|A| = a_1 a_2 ... a_n$.
8. $|A^n| = |A|^n \quad n \in \mathbb{N}$.
9. If $|A| = 0$, then matrix is called singular.

**Singular Matrix**

A square matrix $A$ is said to be singular if $|A| = 0$ and non-singular if $|A| \neq 0$.

1.5 RANK OF MATRIX

The number, $r$ with the following two properties is called the rank of the
matrix

1. There is at least one non-zero minor of order $r$.
2. Every minor of order $(r + 1)$ is zero.

This definition of the rank does uniquely fix the same for, as a consequence
of the condition (2), every minor of order $(r + 2)$, being the sum of multiples
of minors of order $(r + 1)$, will be zero. In fact, every minor of order greater
than \( r \) will be zero as a consequence of the condition (2).
The given following two simple results follow immediately from the definition
1. There exists a non-zero minor of order \( r \), \( \Rightarrow \) the rank is \( \geq r \).
2. All minors of order \( (r+1) \) are zero \( \Rightarrow \) the rank is \( \leq r \).

In each of the two cases above, we assume \( r \) to satisfy only one of two properties (1) or (2) of the rank. The rank of matrix \( A \) represented by symbol \( \rho(A) \).

**Nullity of a Matrix**

If \( A \) is a square matrix of a order \( n \), then \( n - \rho(A) \) is called the nullity of the matrix \( A \) and is denoted by \( N(A) \). Thus a non-singular square matrix of order \( n \) has rank equal to \( n \) and the nullity of such a matrix is equal to zero.

### 1.6 ADJOINT OF A MATRIX

If every element of a square matrix \( A \) be replaced by its cofactor in \( |A| \), then the transpose of the matrix so obtained is called the Adjoint of matrix \( A \) and it is denoted by \( \text{adj } A \). Thus, if \( A = \{a_{ij}\} \) be a square matrix and \( F_{ij} \) be the cofactor of \( a_{ij} \) in \( |A| \), then \( \text{adj } A = \{F_{ij}\} \).

#### PROPERTIES OF ADJOINT MATRIX

If \( A, B \) are square matrices of order \( n \) and \( I_n \) is corresponding unit matrix, then
1. \( A \ (\text{adj } A) = |A| I_n = (\text{adj } A) A \)
2. \( |\text{adj } A| = |A|^{n-1} \)
3. \( \text{adj } (\text{adj } A) = |A|^{n-2} A \)
4. \( |\text{adj } (\text{adj } A)| = |A|^{n(n-1)} \)
5. \( \text{adj } (A^T) = (\text{adj } A)^T \)
6. \( \text{adj } (AB) = (\text{adj } B) (\text{adj } A) \)
7. \( \text{adj } (A^n) = (\text{adj } A)^n, m \in \mathbb{N} \)
8. \( \text{adj } (kA) = k^{n-1}(\text{adj } A), k \in \mathbb{R} \)

### 1.7 INVERSE OF A MATRIX

If \( A \) and \( B \) are two matrices such that \( AB = I = BA \), then \( B \) is called the inverse of \( A \) and it is denoted by \( A^{-1} \). Thus,

\[
A^{-1} = B \iff AB = I = BA
\]

To find inverse matrix of a given matrix \( A \) we use following formula

\[
A^{-1} = \frac{\text{adj } A}{|A|}
\]

Thus \( A^{-1} \) exists if \( |A| \neq 0 \) and matrix \( A \) is called invertible.
PROPERTIES OF INVERSE MATRIX

Let \( A \) and \( B \) are two invertible matrices of the same order, then

1. \( (A^T)^{-1} = (A^{-1})^T \)
2. \( (AB)^{-1} = B^{-1}A^{-1} \)
3. \( (A^k)^{-1} = (A^{-1})^k, k \in \mathbb{N} \)
4. \( \text{adj}(A^{-1}) = (\text{adj} A)^{-1} \)
5. \( |A|^{-1} = \frac{1}{|A|} = |A|^{-1} \)
6. If \( A = \text{diag}(a_1, a_2, ..., a_n) \), then \( A^{-1} = \text{diag}(a_1^{-1}, a_2^{-1}, ..., a_n^{-1}) \)
7. \( AB = AC \Rightarrow B = C \), if \( |A| \neq 0 \)

1.7.1 Elementary Transformations

Any one of the following operations on a matrix is called an elementary transformation (or \( E \)-operation).

1. Interchange of two rows or two columns
   (1) The interchange of \( i \)-th and \( j \)-th rows is denoted by \( R_i \leftrightarrow R_j \)
   (2) The interchange of \( i \)-th and \( j \)-th columns is denoted by \( C_i \leftrightarrow C_j \).

2. Multiplication of (each element) a row or column by a \( k \).
   (1) The multiplication of \( i \)-th row by \( k \) is denoted by \( R_i \rightarrow kR_i \)
   (2) The multiplication of \( i \)-th column by \( k \) is denoted by \( C_i \rightarrow kC_i \)

3. Addition of \( k \) times the elements of a row (or column) to the corresponding elements of another row (or column), \( k \neq 0 \)
   (1) The addition of \( k \) times the \( j \)-th row to the \( i \)-th row is denoted by \( R_i \rightarrow R_i + kR_j \).
   (2) The addition of \( k \) times the \( j \)-th column to the \( i \)-th column is denoted by \( C_i \rightarrow C_i + kC_j \).

   If a matrix \( B \) is obtained from a matrix \( A \) by one or more \( E \)-operations, then \( B \) is said to be equivalent to \( A \). They can be written as \( A \sim B \).

1.7.2 Inverse of Matrix by Elementary Transformations

The elementary row transformations which reduces a square matrix \( A \) to the unit matrix, when applied to the unit matrix, gives the inverse matrix \( A^{-1} \). Let \( A \) be a non-singular square matrix. Then,

\[ A = IA \]

Apply suitable \( E \)-row operations to \( A \) on the left hand side so that \( A \) is reduced to \( I \). Simultaneously, apply the same \( E \)-row operations to the pre-factor \( I \) on right hand side. Let \( I \) reduce to \( B \), so that \( I = BA \). Post-multiplying by \( A^{-1} \), we get

\[ IA^{-1} = BAA^{-1} \]

or \( \quad A^{-1} = B(AA^{-1}) = BI = B \)

or \( \quad B = A^{-1} \)
1.8 ECHELON FORM

A matrix is said to be in Echelon form if,
1. Every row of matrix \(A\) which has all its entries 0 occurs below every row which has a non-zero entry.
2. The first non-zero entry in each non-zero row is equal to one.
3. The number of zeros before the first non-zero element in a row is less than the number of such zeros in the next row.

**Rank of a matrix in the Echelon form**

The rank of a matrix in the echelon form is equal to the number of non-zero rows of the given matrix. For example,

\[
\begin{bmatrix}
0 & 2 & 6 & 1 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 34
\end{bmatrix}
\]

is a 3x4 matrix with rank 2.

1.9 NORMAL FORM

By a finite number of elementary transformations, every non-zero matrix \(A\) of order \(m \times n\) and rank \(r(>0)\) can be reduced to one of the following forms.

\[
\begin{bmatrix}
I_r & 0 \\
0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_r & 0 \\
0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_r & 0 \\
0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_r & 0 \\
0 & 0
\end{bmatrix}
\]

\(I_r\) denotes identity matrix of order \(r\). Each one of these four forms is called Normal Form or Canonical Form or Orthogonal Form.

**Procedure for Reduction of Normal Form**

Let \(A = [a_{ij}]\) be any matrix of order \(m \times n\). Then, we can reduce it to the normal form of the matrix \(A\) by subjecting it to a number of elementary transformation using following methodology.

**METHODOLOGY: REDUCTION OF NORMAL FORM**

1. We first interchange a pair of rows (or columns), if necessary, to obtain a non-zero element in the first row and first column of the matrix \(A\).
2. Divide the first row by this non-zero element, if it is not 1.
3. We subtract appropriate multiples of the elements of the first row from other rows so as to obtain zeroes in the remainder of the first column.
4. We subtract appropriate multiple of the elements of the first column from other columns so as to obtain zeroes in the remainder of the first row.
5. We repeat the above four steps starting with the element in the second row and the second column.
6. Continue this process down the leading diagonal until the end of the diagonal is reached or until all the remaining elements in the matrix are zero.
EXERCISE 1

QUE 1.1

Match the items in column I and II.

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. Singular matrix</td>
<td>1. Determinant is not defined</td>
</tr>
<tr>
<td>Q. Non-square matrix</td>
<td>2. Determinant is always one</td>
</tr>
<tr>
<td>R. Real symmetric</td>
<td>3. Determinant is zero</td>
</tr>
<tr>
<td>S. Orthogonal matrix</td>
<td>4. Eigenvalues are always real</td>
</tr>
<tr>
<td></td>
<td>5. Eigenvalues are not defined</td>
</tr>
</tbody>
</table>

(A) P-3, Q-1, R-4, S-2  
(B) P-2, Q-3, R-4, S-1  
(C) P-3, Q-2, R-5, S-4  
(D) P-3, Q-4, R-2, S-1

QUE 1.2

Cayley-Hamilton Theorem states that a square matrix satisfies its own characteristic equation. Consider a matrix

$$A = \begin{pmatrix} -3 & 2 \\ -2 & 0 \end{pmatrix}$$

If A be a non-zero square matrix of orders n, then

(A) the matrix $A + A'$ is anti-symmetric, but the matrix $A - A'$ is symmetric

(B) the matrix $A + A'$ is symmetric, but the matrix $A - A'$ is anti-symmetric

(C) Both $A + A'$ and $A - A'$ are symmetric

(D) Both $A + A'$ and $A - A'$ are anti-symmetric

QUE 1.3

If $A$ and $B$ are two odd order skew-symmetric matrices such that $AB = BA$, then what is the matrix $AB$?

(A) An orthogonal matrix

(B) A skew-symmetric matrix

(C) A symmetric matrix

(D) An identity matrix

QUE 1.4

If $A$ and $B$ are matrices of order $4 \times 4$ such that $A = 5B$ and $|A| = \alpha|B|$, then $\alpha$ is ____________.
QUE 1.5 If the rank of a \((5 \times 6)\) matrix \(A\) is 4, then which one of the following statements is correct?
(A) \(A\) will have four linearly independent rows and four linearly independent columns
(B) \(A\) will have four linearly independent rows and five linearly independent columns
(C) \(AA^T\) will be invertible
(D) \(A^T A\) will be invertible

QUE 1.6 If \(A_{n \times n}\) is a triangular matrix then \(\det A\) is
(A) \(\prod_{i=1}^{n} (-1)^{a_{ii}}\)
(B) \(\prod_{i=1}^{n} a_{ii}\)
(C) \(\sum_{i=1}^{n} (-1)^{a_{ii}}\)
(D) \(\sum_{i=1}^{n} a_{ii}\)

QUE 1.7 If \(A \in \mathbb{R}_{n \times n}, \det A \neq 0\), then \(A\) is
(A) non singular and the rows and columns of \(A\) are linearly independent.
(B) non singular and the rows \(A\) are linearly dependent.
(C) non singular and the \(A\) has one zero rows.
(D) singular

QUE 1.8 Square matrix \(A\) of order \(n\) over \(\mathbb{R}\) has rank \(n\). Which one of the following statement is not correct?
(A) \(A^T\) has rank \(n\)
(B) \(A\) has \(n\) linearly independent columns
(C) \(A\) is non-singular
(D) \(A\) is singular

QUE 1.9 Determinant of the matrix \[
\begin{bmatrix}
5 & 3 & 2 \\
1 & 2 & 6 \\
3 & 5 & 10
\end{bmatrix}
\] is __________

QUE 1.10 The value of the determinant \[
\begin{vmatrix}
a & h & g \\
h & b & f \\
g & f & c
\end{vmatrix}
\] is
(A) \(abc + 2fg - af^2 - bg^2 - ch^2\)
(B) \(ab + a + c + d\)
(C) \(abc + ab - bc - cg\)
(D) \(a + b + c\)

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QUE 1.11: The value of the determinant\[
\begin{vmatrix}
67 & 19 & 21 \\
39 & 13 & 14 \\
81 & 24 & 26
\end{vmatrix}
\] is ________

QUE 1.12: If \[
\begin{vmatrix}
1 & 3 & 2 \\
0 & 5 & -6 \\
2 & 7 & 8
\end{vmatrix}
= 26,
\] then the determinant of the matrix \[
\begin{vmatrix}
0 & 5 & -6 \\
1 & 3 & 2
\end{vmatrix}
\] is ________

QUE 1.13: The determinant of the matrix \[
\begin{vmatrix}
0 & 1 & 0 & 2 \\
-1 & 1 & 1 & 3 \\
0 & 0 & 0 & 1 \\
1 & -2 & 0 & 1
\end{vmatrix}
\] is ________

QUE 1.14: Let \( A \) be an \( m \times n \) matrix and \( B \) an \( n \times m \) matrix. It is given that
\[
\text{determinant } (I_n + AB) = \text{determinant } (I_n + BA),
\] where \( I_k \) is the \( k \times k \) identity matrix. Using the above property, the determinant of the matrix given below is ________
\[
\begin{vmatrix}
2 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 \\
1 & 1 & 2 & 1 \\
1 & 1 & 1 & 2
\end{vmatrix}
\]

QUE 1.15: Let \( A = \begin{bmatrix} 3 & 1 - 2i \\ 1 - 2i & 2 \end{bmatrix} \), then
\[
(1) \quad A = \begin{bmatrix} 3 & 1 - 2i \\ 1 + 2i & 2 \end{bmatrix} \\
(2) \quad A^* = \begin{bmatrix} 2 & 1 + 2i \\ 1 - 2i & 2 \end{bmatrix} \\
(3) \quad A^* = A \\
(4) \quad A \text{ is hermitian matrix}
\]
Which of above statement is/are correct?
(A) 1 and 3 \\
(B) 1, 2 and 3 \\
(C) 1 and 4 \\
(D) All are correct

QUE 1.16: For which value of \( \lambda \) will the matrix given below become singular?
\[
\begin{vmatrix}
8 & \lambda & 0 \\
4 & 0 & 2 \\
12 & 6 & 0
\end{vmatrix}
\]

QUE 1.17: If \[
\begin{vmatrix}
0 & 1 & -2 \\
-1 & 0 & 3 \\
2 & -2 & \lambda
\end{vmatrix}
\] is a singular, then \( \lambda \) is ________
**QUE 1.18** Multiplication of matrices \( \mathbf{E} \) and \( \mathbf{F} \) is \( \mathbf{G} \). Matrices \( \mathbf{E} \) and \( \mathbf{G} \) are
\[
\mathbf{E} = \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta \\
0 & 0 & 1
\end{bmatrix}
\quad \text{and} \quad
\mathbf{G} = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 & 1
\end{bmatrix}
\]
What is the matrix \( \mathbf{F} \)?

(A) \[
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta \\
0 & 0 & 1
\end{bmatrix}
\]

(B) \[
\begin{bmatrix}
-\cos \theta & \sin \theta \\
\sin \theta & -\cos \theta \\
0 & 0 & 1
\end{bmatrix}
\]

(C) \[
\begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta \\
0 & 0 & 1
\end{bmatrix}
\]

(D) \[
\begin{bmatrix}
\sin \theta & -\cos \theta \\
\cos \theta & \sin \theta \\
0 & 0 & 1
\end{bmatrix}
\]

---

**QUE 1.19** Rank of matrix \[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
-2 & 0 & 5 & 7
\end{bmatrix}
\]
is

---

**QUE 1.20** The rank of the matrix \[
\begin{bmatrix}
1 & 1 & 1 \\
1 & -1 & 0 \\
1 & 1 & 1
\end{bmatrix}
\]
is

---

**QUE 1.21** Given, \[
\mathbf{A} = \begin{bmatrix}
1 & 2 & 3 \\
1 & 4 & 2 \\
2 & 6 & 5
\end{bmatrix}
\]

(1) \( |\mathbf{A}| = 0 \)
(2) \( |\mathbf{A}| \neq 0 \)
(3) \( \text{rank}(\mathbf{A}) = 2 \)
(4) \( \text{rank}(\mathbf{A}) = 3 \)
Which of above statement is/are correct?
(A) 1, 3 and 4
(B) 1 and 3
(C) 1, 2 and 4
(D) 2 and 4

---

**QUE 1.22** Given, \[
\mathbf{A} = \begin{bmatrix}
2 & 1 & -1 \\
0 & 3 & -2 \\
2 & 4 & -3
\end{bmatrix}
\]

(1) \( |\mathbf{A}| = 0 \)
(2) \( |\mathbf{A}| \neq 0 \)
(3) \( \text{rank}(\mathbf{A}) = 2 \)
(4) \( \text{rank}(\mathbf{A}) = 5 \)
Which of above statement is/are correct?
(A) 1, 3 and 4
(B) 1 and 3
(C) 1, 2 and 4
(D) 2 and 4

---

**QUE 1.23** Given matrix \[
\mathbf{A} = \begin{bmatrix}
4 & 2 & 1 & 3 \\
6 & 3 & 4 & 7 \\
2 & 1 & 0 & 1
\end{bmatrix}
\]
the rank of the matrix is

---

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QUE 1.24  The rank of the matrix \( A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 7 & \lambda \\ 1 & 4 & 5 \end{bmatrix} \) is 2. The value of \( \lambda \) must be

QUE 1.25  The rank of matrix \( \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \) is - - - - - - - -

QUE 1.26  Given,
\[
A = \begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}
\]
(1) \( |A| = 0 \)  
(2) Two rows are identical  
(3) rank \( (A) = 2 \)  
(4) rank \( (A) = 3 \)
Which of above statement is/are correct?
(A) 1, 3 and 4  
(B) 1 and 3  
(C) 1, 2 and 3  
(D) 2 and 4

QUE 1.27  Two matrices \( A \) and \( B \) are given below:
\[
A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}, \quad B = \begin{bmatrix} p^2 + q^2 & pr + qs \\ pr + qs & r^2 + s^2 \end{bmatrix}
\]
If the rank of matrix \( A \) is \( N \), then the rank of matrix \( B \) is
(A) \( N/2 \)  
(B) \( N - 1 \)  
(C) \( N \)  
(D) \( 2N \)

QUE 1.28  If \( x, y, z \) are in AP with common difference \( d \) and the rank of the matrix
\[
\begin{bmatrix} 4 & 5 & x \\ 5 & 6 & y \\ 6 & k & z \end{bmatrix}
\]
is 2, then the value of \( d \) and \( k \) are
(A) \( d = x/2; k \) is an arbitrary number  
(B) \( d \) an arbitrary number; \( k = 7 \)  
(C) \( d = k; k = 5 \)  
(D) \( d = x/2; k = 6 \)

QUE 1.29  The rank of a \( 3 \times 3 \) matrix \( C = AB \), found by multiplying a non-zero column matrix \( A \) of size \( 3 \times 1 \) and a non-zero row matrix \( B \) of size \( 1 \times 3 \), is
(A) 0  
(B) 1  
(C) 2  
(D) 3
QUE 1.30
If \( A = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} \) and the point \((x_1, y_1), (y_2, y_2), (x_3, y_2)\) are collinear, then the rank of matrix \( A \) is
(A) less than 3 (B) 3 (C) 1 (D) 0

QUE 1.31
Let \( A = [a_{ij}], 1 \leq i, j \leq n \) with \( n \geq 3 \) and \( a_{ij} = i \cdot j \). Then the rank of \( A \) is
(A) 0 (B) 1 (C) \( n - 1 \) (D) \( n \)

QUE 1.32
Let \( P \) be a matrix of order \( m \times n \), and \( Q \) be a matrix of order \( n \times p \), \( n \neq p \).
If \( \rho(P) = n \) and \( \rho(Q) = p \), then \( \rho(PQ) \) is
(A) \( n \) (B) \( p \) (C) \( np \) (D) \( n + p \)

QUE 1.33
\( x = [x_1, x_2, \ldots, x_n]^T \) is an \( n \)-tuple nonzero vector. The \( n \times n \) matrix \( V = xx^T \)
(A) has rank zero (B) has rank 1 (C) is orthogonal (D) has rank \( n \)

QUE 1.34
If \( x, y, z \) in A.P. with common difference \( d \) and the rank of the matrix
\[
\begin{bmatrix}
4 & 5 & x \\
5 & 6 & y \\
6 & k & z
\end{bmatrix}
\]
is 2, then the values of \( d \) and \( k \) are respectively
(A) \( \frac{x}{4} \) and 7 (B) 7, and \( \frac{x}{4} \)
(C) \( \frac{x}{7} \) and 5 (D) 5, and \( \frac{x}{7} \)

QUE 1.35
If the rank of a \((5 \times 6)\) matrix \( Q \) is 4, then which one of the following statement is correct?
(A) \( Q \) will have four linearly independent rows and four linearly independent columns
(B) \( Q \) will have four linearly independent rows and five linearly independent columns
(C) \( QQ^T \) will be invertible
(D) \( Q^T Q \) will be invertible
The adjoint matrix of \( \begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix} \) is

(A) \( \begin{bmatrix} 4 & 2 \\ 0 & 1 \end{bmatrix} \)  
(B) \( \begin{bmatrix} 1 & 0 \\ 4 & 2 \end{bmatrix} \)  
(C) \( \begin{bmatrix} 2 & 4 \\ 1 & 0 \end{bmatrix} \)  
(D) \( \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} \)

If \( A = \begin{bmatrix} x & y \\ z & b \end{bmatrix} \), then \( \text{adj(adj}(A)) \) is equal to

(A) \( \begin{bmatrix} b & -z \\ -y & x \end{bmatrix} \)  
(B) \( \begin{bmatrix} b & z \\ y & x \end{bmatrix} \)  
(C) \( \begin{bmatrix} 1 \\ b \end{bmatrix} \begin{bmatrix} x & y \end{bmatrix} \)  
(D) None of these

If \( A \) is a \( 3 \times 3 \) matrix and \( |A| = 2 \) then \( A \) (adj A) is equal to

(A) \( \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \)  
(B) \( \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \)  
(C) \( \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \)  
(D) None of these

If \( A \) is a \( 2 \times 2 \) non-singular square matrix, then \( \text{adj(adj}(A)) \) is

(A) \( A^2 \)  
(B) \( A \)  
(C) \( A^{-1} \)  
(D) None of the above

Common Data For Q. 40 to 42

If \( A \) is a 3 - rowed square matrix such that \( |A| = 3 \).

The adj(adj A) is equal to

(A) \( 3A \)  
(B) \( 9A \)  
(C) \( 27A \)  
(D) \( 81A \)

The value of |adj(adj A)| is equal to

(A) 3  
(B) 9  
(C) 27  
(D) 81

The value of \( |\text{adj(adj}(A)^2)| \) is equal to

(A) \( 3^4 \)  
(B) \( 3^8 \)  
(C) \( 3^{16} \)  
(D) \( 3^{32} \)
QUE 1.43 The rank of an \( n \) row square matrix \( A \) is \((n - 1)\), then

(A) \( \text{adj} A \neq 0 \)
(B) \( \text{adj} A = 0 \)
(C) \( \text{adj} A = I_n \)
(D) \( \text{adj} A = I_{n-1} \)

QUE 1.44 The adjoint of matrix \( A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & 1 \end{bmatrix} \) is equal to

(A) \( A \)
(B) \( 3A \)
(C) \( 3A^T \)
(D) \( A^T \)

QUE 1.45 The matrix, that has an inverse is

(A) \( \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix} \)
(B) \( \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \)
(C) \( \begin{bmatrix} 6 & 2 \\ 9 & 3 \end{bmatrix} \)
(D) \( \begin{bmatrix} 8 & 2 \\ 4 & 1 \end{bmatrix} \)

QUE 1.46 The inverse of the matrix \( A = \begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix} \) is

(A) \( \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix} \)
(B) \( \begin{bmatrix} 5 & -3 \\ 3 & 1 \end{bmatrix} \)
(C) \( \begin{bmatrix} -5 & -2 \\ -3 & -1 \end{bmatrix} \)
(D) \( \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix} \)

QUE 1.47 The inverse of the \( 2 \times 2 \) matrix \( \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix} \) is

(A) \( \begin{bmatrix} 1 & -2 \\ 3 & -5 \end{bmatrix} \)
(B) \( \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \)
(C) \( \begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix} \)
(D) \( \begin{bmatrix} 1 & -2 \\ -3 & -5 \end{bmatrix} \)

QUE 1.48 If \( B \) is an invertible matrix whose inverse in the matrix \( \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} \), then \( B \) is

(A) \( \begin{bmatrix} 6 & -4 \\ -5 & 6 \end{bmatrix} \)
(B) \( \begin{bmatrix} 1 & 4 \\ 5 & 6 \end{bmatrix} \)
(C) \( \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} \)
(D) \( \begin{bmatrix} 1 & 4 \\ 5 & 6 \end{bmatrix} \)

QUE 1.49 Matrix \( M = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \) is an orthogonal matrix. The value of \( |B| \) is

(A) \( \frac{1}{2} \)
(B) \( \frac{1}{\sqrt{2}} \)
(C) 1
(D) 0

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QUE 1.50 If \( A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \) then \( A^{-1} \) is

(A) \( \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} \)
(B) \( \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix} \)
(C) \( \begin{bmatrix} 3 & 2 \\ 2 & 5 \end{bmatrix} \)
(D) Undefined

QUE 1.51 The inverse of matrix \( A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ 3 & 1 & 2 \end{bmatrix} \) is equal to

(A) \( \begin{bmatrix} 2 & 0 & 0 \\ -5 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix} \)
(B) \( \begin{bmatrix} 2 & 0 & 0 \\ -5 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix} \)
(C) \( \begin{bmatrix} 1 & 0 & 0 \\ -10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix} \)
(D) \( \begin{bmatrix} 1 & 0 & 0 \\ -10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix} \)

QUE 1.52 If \( \det A = 7 \), where \( A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & b & c \end{bmatrix} \) then \( \det(2A)^{-1} \) is______

QUE 1.53 If \( R = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \end{bmatrix} \), the top of \( R^{-1} \) is

(A) [5, 6, 4]  
(B) [5, -3, 1]  
(C) [2, 0, -1]  
(D) [2, -1, \frac{1}{2}]

QUE 1.54 Let \( B \) be an invertible matrix and inverse of \( 7B \) is \( \begin{bmatrix} -1 & 2 \\ 4 & -7 \end{bmatrix} \) the matrix \( B \) is

(A) \( \begin{bmatrix} 1 & \frac{1}{7} \\ \frac{4}{7} & \frac{1}{7} \end{bmatrix} \)
(B) \( \begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix} \)
(C) \( \begin{bmatrix} 1 & \frac{4}{7} \\ \frac{2}{7} & \frac{1}{7} \end{bmatrix} \)
(D) \( \begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix} \)

QUE 1.55 If \( A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 0 \\ 1 & 4 & 0 \end{bmatrix} \), then \( \det(A^{-1}) \) is equal to______
**QUE 1.56**

Given an orthogonal matrix \( A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \), \( [A^T A]^{-1} \) is

\[
\begin{bmatrix}
\frac{1}{2} & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & \frac{1}{2}
\end{bmatrix}
\]

\( A \)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\( B \)

\[
\begin{bmatrix}
\frac{1}{2} & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & \frac{1}{2}
\end{bmatrix}
\]

\( C \)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\( D \)

**QUE 1.57**

Given an orthogonal matrix

\[
A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}
\]

\( [A^T A]^{-1} \) is

\[
\begin{bmatrix}
\frac{1}{2} & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & \frac{1}{2}
\end{bmatrix}
\]

\( A \)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\( B \)

\[
\begin{bmatrix}
\frac{1}{2} & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & \frac{1}{2}
\end{bmatrix}
\]

\( C \)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\( D \)

**QUE 1.58**

A is \( m \times n \) full rank matrix with \( m > n \) and \( I \) is identity matrix. Let matrix \( \bar{A} = (A^T)^{-1} A^{-1} \). Then, which one of the following statement is FALSE?

(A) \( A \bar{A} \bar{A} = A \)

(B) \( (A \bar{A})^2 \)

(C) \( A \bar{A} = I \)

(D) \( A \bar{A} \bar{A} = \bar{A} \)

**QUE 1.59**

For a matrix \( [M] = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{2}{5} & \frac{1}{5} \end{bmatrix} \), the transpose of the matrix is equal to the inverse of the matrix, \( [M]^T = [M]^{-1} \). The value of \( x \) is given by

(A) \( -\frac{4}{5} \)

(B) \( -\frac{3}{5} \)

(C) \( \frac{3}{5} \)

(D) \( \frac{4}{5} \)

**QUE 1.60**

If \( A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix} \) and \( A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \) then the value of \( x \) is...
QUE 1.61
The value of \[
\begin{bmatrix}
 2 & -1 \\
 3 & 2
\end{bmatrix}
\] is
(A) \[
\begin{bmatrix}
 8 \\
 3
\end{bmatrix}
\]  
(B) \[
\begin{bmatrix}
 3 \\
 8
\end{bmatrix}
\]  
(C) \[-3, -8\]  
(D) \[3, 8\]

QUE 1.62
If \[
A = \begin{bmatrix}
 1 & 2 & 0 \\
 3 & -1 & 4
\end{bmatrix}
\] then \(AA^T\) is
(A) \[
\begin{bmatrix}
 1 & 3 \\
 -1 & 4
\end{bmatrix}
\]  
(B) \[
\begin{bmatrix}
 1 & 0 & 1 \\
 0 & -1 & 2 \\
 3 & 1 & 3
\end{bmatrix}
\]  
(C) \[
\begin{bmatrix}
 5 & 1 \\
 1 & 26
\end{bmatrix}
\]  
(D) Undefined

QUE 1.63
If \[
A = \begin{bmatrix}
 -2 & 1 \\
 3 & 5
\end{bmatrix}
\] then the matrix \(A\) is equal to
(A) \[
\begin{bmatrix}
 1 & 2 \\
 3 & 5
\end{bmatrix}
\]  
(B) \[
\begin{bmatrix}
 2 & 1 \\
 5 & 3
\end{bmatrix}
\]  
(C) \[
\begin{bmatrix}
 5 & 3 \\
 2 & 1
\end{bmatrix}
\]  
(D) Undefined

QUE 1.64
Let, \[
A = \begin{bmatrix}
 2 & 0.1 \\
 0 & 3
\end{bmatrix}
\] and \(A^{-1} = 1/a\begin{bmatrix}
 1 & 0 \\
 0 & 1
\end{bmatrix}\). Then \((a + b) = \ldots\)

QUE 1.65
Let \[
A = \begin{bmatrix}
 2 & 0.1 \\
 0 & 3
\end{bmatrix}
\] and \(A^{-1} = 1/a\begin{bmatrix}
 1 & 0 \\
 0 & 1
\end{bmatrix}\). Then \((a + b)\) is
(A) \[
\frac{7}{20}
\]  
(B) \[
\frac{3}{20}
\]  
(C) \[
\frac{19}{20}
\]  
(D) \[
\frac{11}{20}
\]

QUE 1.66
If \[
A = \begin{bmatrix}
 2 & 6 \\
 3 & 9
\end{bmatrix}
\] and \[
B = \begin{bmatrix}
 3 & x \\
 y & 2
\end{bmatrix}
\] then in order that \(AB = 0\), the values of \(x\) and \(y\) will be respectively
(A) \(-6\) and \(-1\)  
(B) \(6\) and \(1\)  
(C) \(6\) and \(-3\)  
(D) \(5\) and \(14\)

QUE 1.67
If \[
A = \begin{bmatrix}
 1 & 1 & 0 \\
 1 & 0 & 1
\end{bmatrix}
\] and \[
B = \begin{bmatrix}
 1 & 0 \\
 1 & 0 \\
 1 & 0
\end{bmatrix}
\] the product of \(A\) and \(B\) is
(A) \[
\begin{bmatrix}
 1 \\
 1 \\
 1
\end{bmatrix}
\]  
(B) \[
\begin{bmatrix}
 1 & 0 \\
 0 & 1 \\
 0 & 2
\end{bmatrix}
\]  
(C) \[
\begin{bmatrix}
 1 & 0 \\
 0 & 1 \\
 0 & 2
\end{bmatrix}
\]  
(D) \[
\begin{bmatrix}
 1 & 0 \\
 0 & 1 \\
 0 & 2
\end{bmatrix}
\]
QUE 1.68
If \( A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} \), then \((AB)^T\) is

(A) \( \begin{bmatrix} 1 & 4 \\ 4 & 4 \end{bmatrix} \)  
(B) \( \begin{bmatrix} 1 & 4 \\ 1 & 4 \end{bmatrix} \)  
(C) \( \begin{bmatrix} 1 & 4 \\ 1 & 4 \end{bmatrix} \)  
(D) \( \begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix} \)

QUE 1.69
If \( A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix} \), then \( AB \) is

(A) \( \begin{bmatrix} -1 & -8 & -10 \\ 9 & 22 & 15 \\ -1 & -8 & -10 \end{bmatrix} \)  
(B) \( \begin{bmatrix} 0 & 0 & -10 \\ -1 & -2 & -5 \\ 0 & 21 & -15 \end{bmatrix} \)  
(C) \( \begin{bmatrix} 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix} \)  
(D) \( \begin{bmatrix} 0 & -10 \\ 0 & 8 & -10 \end{bmatrix} \)

QUE 1.70
If \( X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \), then the rank of \( X^T X \), where \( X^T \) denotes the transpose of \( X \), is \( \underline{4} \).

QUE 1.71
Consider the matrices \( X \) \((4 \times 3)\), \( Y \) \((4 \times 3)\) and \( P \) \((2 \times 3)\). The order of \( [P(X^T Y)P^T]^T \) will be

(A) \((2 \times 2)\)  
(B) \((3 \times 3)\)  
(C) \((4 \times 3)\)  
(D) \((3 \times 4)\)

QUE 1.72
If \( A_\alpha = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \), then consider the following statements :

1. \( A_\alpha A_\beta = A_{\alpha + \beta} \)  
2. \( A_\alpha A_\beta = A_{\alpha + \beta} \)  
3. \( (A_\alpha)^n = \begin{bmatrix} \cos^n \alpha & \sin^n \alpha \\ -\sin^n \alpha & \cos^n \alpha \end{bmatrix} \)  
4. \( (A_\alpha)^n = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \)

Which of the above statements are true ?

(A) 1 and 2  
(B) 2 and 3  
(C) 2 and 4  
(D) 3 and 4

QUE 1.73
If \( A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix} \), then \((I - A) \begin{bmatrix} \cos \frac{\alpha}{2} & -\sin \frac{\alpha}{2} \\ \sin \frac{\alpha}{2} & \cos \frac{\alpha}{2} \end{bmatrix} \) is equal to

(A) \( I - 2A \)  
(B) \( I - A \)  
(C) \( I + 2A \)  
(D) \( I + A \)
QUE 1.74
Let \( \mathbf{A} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \)

(1) \( \mathbf{A}\mathbf{A}^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) 
(2) \( |\mathbf{A}\mathbf{A}^T| = 1 \)
(3) \( \mathbf{A} \) is orthogonal matrix 
(4) \( \mathbf{A} \) is not a orthogonal matrix

Which of above statement is/are correct?
(A) 1, 3 and 4
(B) 2 and 3
(C) 1, 2 and 3
(D) 2 and 4

QUE 1.75
If the product of matrices
\[
\mathbf{A} = \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} \cos^2 \phi & \cos \phi \sin \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{bmatrix}
\]
is a null matrix, then \( \theta \) and \( \phi \) differ by
(A) an even multiple \( \frac{\pi}{2} \)
(B) an even multiple \( \pi \)
(C) an odd multiple of \( \frac{\pi}{2} \)
(D) an odd multiple of \( \pi \)

QUE 1.76
For a given \( 2 \times 2 \) matrix \( \mathbf{A} \), it is observed that \( \mathbf{A} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \) and \( \mathbf{A} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \). The matrix \( \mathbf{A} \) is

(A) \( \mathbf{A} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \)
(B) \( \mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -2 \end{bmatrix} \)
(C) \( \mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -2 \end{bmatrix} \)
(D) \( \mathbf{A} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \)

QUE 1.77
If \( \mathbf{A} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \), then for every positive integer \( n \), \( \mathbf{A}^n \) is equal to

(A) \( \begin{bmatrix} 1 + 2n & 4n \\ n & 1 + 2n \end{bmatrix} \)
(B) \( \begin{bmatrix} 1 - 2n & -4n \\ n & 1 + 2n \end{bmatrix} \)
(C) \( \begin{bmatrix} 1 - 2n & 4n \\ n & 1 + 2n \end{bmatrix} \)
(D) \( \begin{bmatrix} 1 + 2n & -4n \\ n & 1 - 2n \end{bmatrix} \)
QUE 1.78  
For which values of the constants \( b \) and \( c \) is the vector \( \begin{bmatrix} 3 \\ b \\ c \end{bmatrix} \) a linear combination of \( \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \) and \( \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix} \)?

(A) 9, 6  
(B) 6, 9  
(C) 6, 6  
(D) 9, 9

---

QUE 1.79  
The values of non-zero numbers \( a, b, c, d, e, f, g, h \) such that the matrix \( \begin{bmatrix} a & b & c \\ d & k & e \\ f & g & h \end{bmatrix} \) is invertible for all real numbers \( k \).

(A) finite solution  
(B) infinite solution  
(C) 0  
(D) none

************
SOL 1.1 Correct option is (A).

(P) Singular Matrix → Determinant is zero \(|A| = 0\)

(Q) Non-square matrix → An \(m \times n\) matrix for which \(m \neq n\), is called non-square matrix. Its determinant is not defined.

(R) Real Symmetric Matrix → Eigen values are always real.

(S) Orthogonal Matrix → A square matrix \(A\) is said to be orthogonal if \(AA^T = I\). Its determinant is always one.

SOL 1.2 Correct option is (B).

Here, if \(A\) be a non-zero square matrix of order \(n\), then the matrix \(A + A^T\) is symmetric, but \(A - A^T\) will be anti-symmetric.

SOL 1.3 Correct option is (C).

If \(A\) and \(B\) are both order skew-symmetric matrices, then

\[ A = -A^T \quad \text{and} \quad B = -B^T \quad \ldots (1) \]

Also, given that \(AB = BA\)

\[ = (-B)^T (-A^T) \quad \text{[from Eq. (1)]} \]

\[ = B^T A^T = (AB)^T \quad \text{i.e., } AB \text{ is a symmetric matrix} \]

SOL 1.4 Correct answer is 625.

If \(k\) is a constant and \(A\) is a square matrix of order \(n \times n\) then \(|kA| = k^n|A|\).

\[ A = 5B \Rightarrow |A| = |5B| = 5^n|B| = 625|B| \]

or

\[ \alpha = 625 \]

SOL 1.5 Correct option is (A).

If rank of \((5 \times 6)\) matrix is 4, then surely it must have exactly 4 linearly independent rows as well as 4 linearly independent columns.

Hence, Rank = Row rank = Column rank

SOL 1.6 Correct option is (B).

From linear algebra for \(A_{n \times n}\) triangular matrix \(\det A\) is equal to the product of the diagonal entries of \(A\).
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SOL 1.7 Correct option is (B).
If \( \det \mathbf{A} \neq 0 \), then \( A_{n \times n} \) is non-singular, but if \( A_{n \times n} \) is non-singular, then no row can be expressed as a linear combination of any other. Otherwise \( \det \mathbf{A} = 0 \)

SOL 1.8 Correct option is (D).
Since, if \( \mathbf{A} \) is a square matrix of rank \( n \), then it cannot be a singular.

SOL 1.9 Correct answer is \(-28\).
\[
\begin{vmatrix}
5 & 3 & 2 \\
1 & 2 & 6 \\
3 & 5 & 10 \\
\end{vmatrix}
= 5(20 - 30) - 3(10 - 18) + 2(5 - 6)
= -50 + 24 - 2 = -28
\]

SOL 1.10 Correct option is (A).
\[
\begin{vmatrix}
a & h & g \\
h & b & f \\
g & f & c \\
\end{vmatrix}
= \begin{vmatrix}
a & f & c \\
h & g & c \\
g & h & f \\
\end{vmatrix} = a(hc - f^2) - h(hc - fg) + g(hf - gb)
= abc - af^2 - h^2c + hfg + gfh - g^2b
= abc + 2fgh - af^2 - bg^2 - ch^2
\]

SOL 1.11 Correct answer is \(-43\).
\[
\text{Determinant} = 67 \begin{vmatrix}
13 & 14 \\
24 & 26 \\
\end{vmatrix} - 19 \begin{vmatrix}
39 & 14 \\
81 & 26 \\
\end{vmatrix} + 21 \begin{vmatrix}
39 & 13 \\
81 & 24 \\
\end{vmatrix}
= 134 + 2280 - 2457 = -43
\]

SOL 1.12 Correct answer is 26.
By interchanging any row or column, the value of determinant will remain same. For the given matrix, the first and third row are interchanged, thus the value remains the same.

SOL 1.13 Correct answer is \(-1\).
We have
\[
\mathbf{A} = \begin{vmatrix}
0 & 1 & 0 & 2 \\
-1 & 1 & 1 & 3 \\
0 & 0 & 0 & 1 \\
1 & -2 & 0 & 1 \\
\end{vmatrix}
\]
Expanding cofactor of \( a_{34} \)
\[
|\mathbf{A}| = -1 \begin{vmatrix}
1 & 0 \\
-1 & 1 \\
\end{vmatrix}
= -[0 - 1(0 - 1) + 0] = -1
\]

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SOL 1.14

Correct answer is 5.

Consider the given matrix be

\[ \mathbf{I}_m + \mathbf{A}\mathbf{B} = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \]

where \( m = 4 \) so, we obtain

\[ \mathbf{A}\mathbf{B} = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \]

Hence, we get

\[ \mathbf{A} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \]

Therefore,

\[ \mathbf{B}\mathbf{A} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = [4] \]

From the given property,

\[ \text{Det} (\mathbf{I}_m + \mathbf{A}\mathbf{B}) = \text{Det}(\mathbf{I}_n + \mathbf{B}\mathbf{A}) \]

\[ \text{Det} \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} = \text{Det} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + [4] \]

\[ = \text{Det} [5] = 5 \]

NOTE:
Determinant of identity matrix is always 1.

SOL 1.15

Correct option is (D).

\[ \overline{\mathbf{A}} = \text{conjugate of } \mathbf{A} = \begin{bmatrix} 3 & 1+2i \\ 1+2i & 2 \end{bmatrix} \]

and \( \mathbf{A}^* = (\overline{\mathbf{A}})^T = \text{transpose of } \overline{\mathbf{A}} = \begin{bmatrix} 3 & 1-2i \\ 1+2i & 2 \end{bmatrix} \]

Since, \( \mathbf{A}^* = \mathbf{A} \)

Hence, \( \mathbf{A} \) is hermitian matrix.

SOL 1.16

Correct answer is 4.

For singularity of matrix,
Matrix Algebra

$\begin{bmatrix} 8 & 0 & 12 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix} = 0$

$8(0 - 12) - \lambda(0 - 2 \times 12) = 0 \Rightarrow \lambda = 4$

**SOL 1.17**
Correct answer is $-2$.
Matrix $A$ is singular if $|A| = 0$

$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -2 & \lambda \end{bmatrix} = 0$

or

$-(-1)^3 \begin{vmatrix} 1 & -2 \\ -2 & \lambda \end{vmatrix} + \begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} + 0 \begin{vmatrix} 0 & 3 \\ 2 & -2 \lambda \end{vmatrix} = 0$

or

$(\lambda - 4) + 2(3) = 0$

or

$\lambda = -2$

**SOL 1.18**
Correct option is (C).
Given $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

We know that the multiplication of a matrix and its inverse be a identity matrix $A A^{-1} = I$

So, we can say that $F$ is the inverse matrix of $E$

$F = E^{-1} = \frac{\text{adj} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}}{|E|}$

$\text{adj} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$

$|E| = \cos \theta \times (\cos \theta - 0) - (-\sin \theta \times (\sin \theta - 0)) + 0$

$= \cos^2 \theta + \sin^2 \theta = 1$

Hence,

$F = \frac{\text{adj} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}}{|E|} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$

$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

**SOL 1.19**
Correct answer is 2.
We have $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 5 & 7 \end{bmatrix}$

It is a $2 \times 4$ matrix, thus $\rho(A) \leq 2$

The second order minor

$\begin{vmatrix} 1 & 2 \\ -2 & 0 \end{vmatrix} = 4 \neq 0$

Hence, $\rho(A) = 2$
SOL 1.20
Correct answer is 2.

We have

\[
A = \begin{bmatrix}
1 & 1 & 1 \\
1 & -1 & 0 \\
1 & 1 & 1 \\
\end{bmatrix}
\]

\[
R_3 - R_1
\]

Since one full row is zero, \( \rho(A) < 3 \)

Now

\[
\begin{bmatrix}
1 & 1 \\
1 & -1 \\
\end{bmatrix}
\]

\(-2 \neq 0, \text{ thus } \rho(A) = 2\)

SOL 1.21
Correct option is (B).

Here,

\[
A = \begin{bmatrix}
1 & 2 & 3 \\
1 & 4 & 2 \\
2 & 6 & 5 \\
\end{bmatrix}
\]

Performing operation \( R_{31}(-1) \), we get

\[
A \sim \begin{bmatrix}
1 & 2 & 3 \\
1 & 4 & 2 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

By operation \( R_{32}(-1) \), we get

\[
A \sim \begin{bmatrix}
1 & 2 & 3 \\
1 & 4 & 2 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

\[|A| = 0\]

and

\[\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix} \neq 0\]

Rank \( (A) = 2\)

SOL 1.22
Correct option is (B).

Here,

\[|A| = 2(-9 + 8) + 2(-2 + 3) = 0\]

But

\[\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \neq 0\]

Hence, rank \( (A) = 2\)

SOL 1.23
Correct answer is 2.

Consider \( 3 \times 3 \) minors, maximum possible rank is 3.

Now we can obtain

\[
\begin{bmatrix} 2 & 1 & 3 \\ 3 & 4 & 7 \\ 1 & 0 & 1 \end{bmatrix} = 0, \quad \begin{bmatrix} 4 & 1 & 3 \\ 6 & 4 & 7 \\ 2 & 0 & 1 \end{bmatrix} = 0 \quad \text{and} \quad \begin{bmatrix} 4 & 2 & 3 \\ 6 & 3 & 7 \\ 2 & 1 & 1 \end{bmatrix} = 0
\]

Since, all \( 3 \times 3 \) minors are zero. Now, we consider \( 2 \times 2 \) minors

\[
\begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix} = 0, \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = 8 - 3 = 5 \neq 0
\]

Hence, rank = 2

SOL 1.24
Correct answer is 13.

Since \( \rho(A) = 2 < \text{order of matrix} \)
Thus \[ |A| = \begin{vmatrix} 2 & -1 & 3 \\ 4 & 7 & \lambda \\ 1 & 4 & 5 \end{vmatrix} = 0 \]

or \((235 - 4\lambda) + 1(20 - \lambda) + 3(16 - 7) = 0\)

or \(70 - 8\lambda + 20 - \lambda + 27 = 0 = 0\)

or, \(\lambda = 13\)

**SOL 1.25**

Correct answer is 3.

It is a 4 \times 4 matrix, so its rank \(\rho(A) \leq 4\)

We have \[ |A| = \begin{vmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{vmatrix} \]

\[ = \begin{vmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{vmatrix} \]

applying \(R_4 - (R_1 + R_2 + R_3) \rightarrow R_4\)

applying \(R_2 - 2R_1 \rightarrow R_2\)

applying \(R_3 - 3R_1 \rightarrow R_3\)

The only fourth order minor is zero.

Since the third order minor \[ \begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & -3 \\ 0 & -4 & -8 \end{vmatrix} = (1)(-4)(-3) = 12 \neq 0 \]

Therefore its rank is \(\rho(A) = 3\)

**SOL 1.26**

Correct option is (C).

Here, \(|A| = 0\)

All minors of order 3 are zero, since two rows are identical.

The second minor \[ \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \neq 0 \]

Hence, \(\text{Rank } (A) = 2\)

**SOL 1.27**

Correct option is (C).

Given the two matrices,

\[ A = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \]

and \[ B = \begin{bmatrix} p^2 + q^2 & pr + qs \\ pr + qs & r^2 + s^2 \end{bmatrix} \]

To determine the rank of matrix \(A\), we obtain its equivalent matrix using the operation, 
\[ a_2 \leftarrow a_2 - \frac{a_{21}}{a_{11}} a_{12} \]
\[
A = \begin{bmatrix}
p & q \\
0 & s - \frac{r}{p}q
\end{bmatrix}
\]

If \( s - \frac{r}{p}q = 0 \)
or \( ps - rq = 0 \)
then rank of matrix \( A \) is 1, otherwise the rank is 2.

Now, we have the matrix
\[
B = \begin{bmatrix}
p^2 + q^2 & pr + qs \\
pr + qs & r^2 + s^2
\end{bmatrix}
\]

To determine the rank of matrix \( B \), we obtain its equivalent matrix using the operation, \( a_{ii} \leftarrow a_{ii} - \frac{a_{i1}a_{1i}}{a_{11}} \) as
\[
B' = \begin{bmatrix}
p^2 + q^2 & pr + qs \\
0 & (r^2 + s^2) - \frac{(pr + qs)^2}{p^2 + q^2}
\end{bmatrix}
\]

If \( (r^2 + s^2) - \frac{(pr + qs)^2}{p^2 + q^2} = (ps - rq)^2 = 0 \)
or \( ps - rq = 0 \)
then rank of matrix \( B \) is 1, otherwise the rank is 2.

Thus, from the above results, we conclude that
If \( ps - rq = 0 \), then rank of matrix \( A \) and \( B \) is 1.
If \( ps - rq \neq 0 \), then rank of \( A \) and \( B \) is 2.

i.e. the rank of two matrices is always same. If rank of \( A \) is \( N \) then rank of \( B \) also \( N \).

**SOL 1.28**

Correct option is (B).

It is given that \( x, y, z \) are in A.P. with common difference \( d \)
\[
x = x, y = x + d, z = x + 2d
\]

Let
\[
|A| = \begin{vmatrix}
4 & 5 & x \\
5 & 6 & y \\
6 & k & z
\end{vmatrix} = \begin{vmatrix}
4 & 5 & x \\
5 & 6 & x + d \\
6 & k & x + 2d
\end{vmatrix} = \begin{vmatrix}
4 & 5 & x \\
1 & 1 & d \\
1 & k & 6
\end{vmatrix}
\]

Applying \( R_2 - R_1 = R_2 \) and \( R_3 - R_2 = R_3 \)
\[
\begin{vmatrix}
4 & 5 & x \\
1 & 1 & d \\
0 & k - 7 & 0
\end{vmatrix}
\]

\[
|A| = 0 \Rightarrow (k - 7)(4d - x) = 0
\]
\[
d = \frac{x}{4}, k = 7.
\]

**SOL 1.29**

Correct option is (B).

Let
\[
A = [a_1, b_1, c_1], \quad B = [a_2, b_2, c_2]
\]

Let
\[
C = AB
\]
The $3 \times 3$ minor of this matrix is zero and all the $2 \times 2$ minors are also zero. So the rank of this matrix is 1, i.e. $\rho[C] = 1$

**SOL 1.30**

Correct option is (A).

Since all point are collinear,

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Therefore $\rho(A) < 3$

**SOL 1.31**

Correct option is (B).

Let $n = 3$

Then $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

and $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{vmatrix}$

Thus rank if $n = 3$ then $\rho(A) = 1$ so possible answer is (B).

**SOL 1.32**

Correct option is (B).

If $P$ is a matrix of order $m \times n$ and $\rho(P) = n$ then $n \leq m$

In the normal form of $P$ only $n$ rows are non-zero

Now $Q$ is a matrix of order $n \times p$ and $\rho(PQ) = p$ then $p \leq n$ but $p \neq n$ but $p \neq n$ so $p < n$.

In the normal form of $Q$ only $p$ rows are non-zero.

Thus the normal form of $PQ$ only $p$ rows are non-zero.

$$\rho(PQ) = p$$

**SOL 1.33**

Correct option is (D).

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}^T$$

$$\mathbf{V} = \mathbf{xx}^T$$

$$= \begin{bmatrix} x_1 & x_1 \\ x_2 & x_2 \\ \vdots & \vdots \\ x_n & x_n \end{bmatrix}$$

So rank of $V$ is $n$. 

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Correct option is (A).

Given that \(x, y, z\) are in A.P. with common differences \(d\).

Thus
\[
y = x + d,
\]
\[
z = x + 2d
\]

Now
\[
\begin{vmatrix}
4 & 5 & x \\
1 & 1 & d \\
1 & k - 6 & d
\end{vmatrix}
= \frac{4}{1} \begin{vmatrix}
5 & x \\
1 & d \\
6 & k + 2d
\end{vmatrix}
= \frac{4}{1} \begin{vmatrix}
5 & x \\
1 & d \\
k - 6 & d
\end{vmatrix}
\]

Apply \(R_2 - R_1 = R_2\) and \(R_3 - R_2 = R_3\)

\[
\begin{vmatrix}
4 & 5 & x \\
1 & 1 & d \\
0 & k - 7 & 0
\end{vmatrix}
\]

Thus
\[
|A| = (k - 7)(4d - x)
\]

Since \(\rho(A) = 2 <\) order of matrix, Thus \(|A| = 0\) or we get
\[
(k - 7)(4d - x) = 0
\]

or
\[
d = \frac{x}{4}, k = 7
\]

Correct option is (A).

Rank of a matrix is no. of linearly independent rows and columns of the matrix.

Here \(\rho(Q) = 4\)

So \(Q\) will have 4 linearly independent rows and flour independent columns.

Correct option is (D).

We have
\[
A = \begin{bmatrix}
1 & -4 \\
0 & 2
\end{bmatrix}
\]

\[
C_{11} = 2, C_{12} = 0, C_{21} = -(-4) = 4, \text{ and } C_{22} = 1
\]

\[
C = \begin{bmatrix}
2 & 0 \\
4 & 1
\end{bmatrix}
\]

\[
adj(A) = C^T
\]

\[
= \begin{bmatrix}
2 & 4 \\
0 & 1
\end{bmatrix}
\]

Correct option is (D).

\[
A = \begin{bmatrix}
x & y \\
z & b
\end{bmatrix}
\]

\[
adj(A) = \begin{bmatrix}
b & -y \\
-z & x
\end{bmatrix}
\]

\[
adj(adj(A)) = \begin{bmatrix}
x & y \\
z & b
\end{bmatrix}
\]

Correct option is (B).

Since, \(A(adj(A)) = |A|I_n\)
We have \( \mathbf{A} (\text{adj} \mathbf{A}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \)

**SOL 1.39**
Correct option is (B).

We know that
\[
\text{adj}(\text{adj}(\mathbf{A})) = |\mathbf{A}|^{-2} \mathbf{A}
\]
Here \( n = 2 \) so we get
\[
\text{adj}(\text{adj}(\mathbf{A})) = |\mathbf{A}|^{-2} \mathbf{A} = |\mathbf{A}|^2 \mathbf{A}
\]
\[
= |\mathbf{A}| \mathbf{I} \mathbf{A} = \mathbf{A}
\]

**SOL 1.40**
Correct option is (A).

We know that
\[
\text{adj}(\text{adj}(\mathbf{A})) = |\mathbf{A}|^{-2} \mathbf{A}
\]
Putting \( n = 3 \) and \( |\mathbf{A}| = 3 \) we get
\[
\text{adj}(\text{adj}(\mathbf{A})) = |\mathbf{A}|^{-2} \mathbf{A} = |\mathbf{A}|^2 \mathbf{A}
\]
\[
= |\mathbf{A}| \mathbf{I} \mathbf{A} = 3 \mathbf{A}
\]

**SOL 1.41**
Correct option is (D).

We have
\[
|\text{adj}(\text{adj}(\mathbf{A}))| = |\mathbf{A}|^{-3} \mathbf{A}
\]
Putting \( n = 3 \) and \( |\mathbf{A}| = 3 \) we get
\[
|\text{adj}(\text{adj}(\mathbf{A}))| = |\mathbf{A}|^{-3} = 3 \mathbf{A}
\]
\[
= |\mathbf{A}|^2 \mathbf{A} = 3^2 = 81
\]

**SOL 1.42**
Correct option is (B).

Let \( \mathbf{B} = \text{adj} \mathbf{A}^2 \) then \( \mathbf{B} \) is also a \( 3 \times 3 \) matrix.
\[
|\text{adj}(\text{adj} \mathbf{A}^2)| = |\text{adj} \mathbf{B}| = |\mathbf{B}|^{-1} = |\mathbf{B}|^2
\]
\[
= |\text{adj} \mathbf{A}^2|^2
\]
\[
= |\mathbf{A}^2|^{-3} |\mathbf{A}^2| = |\mathbf{A}|^{-3} = 3^8
\]

**SOL 1.43**
Correct option is (A).

Since \( \rho(\mathbf{A}) = n - 1 \), at least one \( (n - 1) \) rowed minor of \( \mathbf{A} \) is non-zero, so at least one minor and therefore the corresponding co-factor is non-zero.

So, \( \text{adj} \mathbf{A} \neq 0 \)

**SOL 1.44**
Correct option is (C).

If \( \mathbf{A} = [a_{ij}]_{n \times n} \) then \( \text{det} \mathbf{A} = [C_{ij}]_{n \times n} \) where \( C_{ij} \) is the cofactor of \( a_{ij} \)

Also \( C_{ij} = (-1)^{i+j} M_{ij} \), where \( M_{ij} \) is the minor of \( a_{ij} \), obtained by leaving the row and the column corresponding to \( a_{ij} \) and then taken the determinant of the remaining matrix.

Now, \( M_{11} \) = minor of \( a_{11} \) i.e. \((-1)^{1+1} \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = -3 \)
Similarly

\[
\begin{align*}
M_{12} &= \begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix} = 6; & \quad M_{13} &= \begin{bmatrix} 2 & 1 \\ 2 & -2 \end{bmatrix} = -6 \\
M_{21} &= \begin{bmatrix} -2 & -2 \\ -2 & 1 \end{bmatrix} = -6; & \quad M_{22} &= \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix} = 3; \\
M_{23} &= \begin{bmatrix} -1 & -2 \\ 2 & -2 \end{bmatrix} = 6; & \quad M_{31} &= \begin{bmatrix} -2 & -2 \\ 1 & -2 \end{bmatrix} = 6; \\
M_{32} &= \begin{bmatrix} -1 & -2 \\ 2 & -2 \end{bmatrix} = 6; & \quad M_{33} &= \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix} = 3.
\end{align*}
\]

\[
C_{11} = (-1)^{1+1}M_{11} = -3; \quad C_{12} = (-1)^{1+2}M_{12} = -6; \quad C_{13} = (-1)^{1+3}M_{13} = -6; \quad C_{21} = (-1)^{2+1}M_{21} = 6; \quad C_{22} = (-1)^{2+2}M_{22} = 3; \quad C_{23} = (-1)^{2+3}M_{23} = -6; \quad C_{31} = (-1)^{3+1}M_{31} = 6; \quad C_{32} = (-1)^{3+2}M_{32} = -6; \quad C_{33} = (-1)^{3+3}M_{33} = 3.
\]

Thus

\[
\text{adj } A = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ -6 & -3 & 3 \end{bmatrix} = 3 \begin{bmatrix} -1 & -2 & -2^T \\ 2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = 3A^T.
\]

**SOL 1.45**

Correct option is (B).

If \(|A|\) is zero, \(A^{-1}\) does not exist and the matrix \(A\) is said to be singular. Except (B) all satisfy this condition.

\[|A| = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} = (5)(1) - (2)(2) = 1\]

**SOL 1.46**

Correct option is (A).

We know \(A^{-1} = \frac{1}{|A|} \text{adj } A\)

Here \(|A| = \begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix} = -1\)

Also, \(\text{adj } A = \begin{bmatrix} -5 & -2 \\ -3 & -1 \end{bmatrix}\)

\[A^{-1} = \begin{bmatrix} 1 & -5 & -2 \\ -1 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}\]

**SOL 1.47**

Correct option is (A).

We know \(A^{-1} = \frac{1}{|A|} \text{adj } A\)

Here \(|A| = \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix} = -3\)

Also, \(\text{adj } A = \begin{bmatrix} 7 & -2 \\ -5 & 1 \end{bmatrix}\)
\[ \mathbf{A}^{-1} = \frac{1}{3} \begin{bmatrix} 7 & -2 \\ -5 & 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -7 & 2 \\ 5 & -1 \end{bmatrix} \]

**SOL 1.48**

Correct option is (C).

Let \( \mathbf{B}^{-1} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} = \mathbf{A} \) and \( \mathbf{B} = \mathbf{A}^{-1} \)

We know \( \mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|} \text{adj} \mathbf{A} \)

Here \( |\mathbf{A}| = \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix} = -2 \)

Also, \( \text{adj} \mathbf{A} = \begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix} \)

\[ \mathbf{A}^{-1} = \frac{1}{-2} \begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix} \]
\[ = \begin{bmatrix} -3 & 2 \\ \frac{5}{2} & -\frac{3}{2} \end{bmatrix} \]

**SOL 1.49**

Correct option is (C).

For orthogonal matrix \( \det \mathbf{M} = 1 \) and \( \mathbf{M}^{-1} = \mathbf{M}^T \),

\[ \mathbf{M}^T = \begin{bmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{B} & 0 \end{bmatrix} \]

\[ = \mathbf{M}^{-1} = \frac{1}{-\mathbf{BC}} \begin{bmatrix} 0 & -\mathbf{B} \\ -\mathbf{C} & \mathbf{A} \end{bmatrix} \]

This implies \( \mathbf{B} = \frac{-\mathbf{C}}{-\mathbf{BC}} \)

or \( \mathbf{B} = \frac{1}{\mathbf{B}} \) or \( \mathbf{B} = \pm 1 \)

**SOL 1.50**

Correct option is (D).

Inverse matrix is defined for square matrix only.

**SOL 1.51**

Correct option is (D).

We know \( \mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|} \text{adj} \mathbf{A} \)

\[ |\mathbf{A}| = \begin{vmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ 3 & 1 & 2 \end{vmatrix} = 4 \neq 0, \]

\[ \text{adj} \mathbf{A} = \begin{bmatrix} 4 & 10 & -10 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix} \]

\[ \mathbf{A}^{-1} = \frac{1}{4} \begin{bmatrix} 4 & 0 & 0 \\ 10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix} \]
SOL 1.52
Correct answer is 0.01786.
\[
\det(2A)^{-1} = \frac{1}{2} \begin{vmatrix} 2A \end{vmatrix} = \frac{1}{2^n |A|}
\]
Since \(A\) is a 3 \times 3 matrix, therefore \(n = 3\) and we have \(|A| = 7\).
\[
\det(2A)^{-1} = \frac{1}{2^3 |A|} = \frac{1}{8 \times 7} = \frac{1}{56}
\]

SOL 1.53
Correct option is (B).
\[
C_{11} = 2 - (-3) = 5 \\
C_{21} = -10 - (-3) = -3 \\
C_{31} = (-1) = 1 \\
|R| = (1)C_{11} + 2C_{21} + 2C_{31} = 5 - 6 + 2 = 1
\]

SOL 1.54
Correct option is (A).
Let \((7B)^{-1} = A = \begin{bmatrix} -1 & 2 \\ 4 & -7 \end{bmatrix}\) and \(7B = A^{-1}\)
We know \(A^{-1} = \frac{1}{|A|} \text{adj} A\)
Here \(|A| = \begin{vmatrix} -1 & 2 \\ 4 & -7 \end{vmatrix} = -1\)
Also, \(\text{adj} A = \begin{bmatrix} -7 & -2 \\ -4 & -1 \end{bmatrix}\)
\[
A^{-1} = \begin{bmatrix} -7 & -2 \\ -4 & -1 \end{bmatrix}
\]
or
\[
7B = A^{-1} = \begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix}
\]
or
\[
B = \begin{bmatrix} 7 \\ 4 \end{bmatrix}
\]

SOL 1.55
Correct answer is 0.5.
We have \(A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 3 & 2 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}\)
\[
|A| = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 3 & 2 & 0 \\ 1 & 4 & 1 & 0 \end{vmatrix} = 1(0 - 2) = 2
\]
Now \(\det(A^{-1}) = \frac{1}{\det A} = \frac{1}{2}\)

SOL 1.56
Correct option is (C).
For orthogonal matrix we know that...
\[ AA^T = I \]

and \[ [AA^T]^{-1} = I^{-1} = I \]

**SOL 1.57**

Correct option is (C).

From orthogonal matrix
\[ [AA^T] = I \]

Since the inverse of \( I \) is \( I \), thus
\[ [AA^T]^{-1} = I^{-1} = I \]

**SOL 1.58**

Correct option is (D).

\[ A' = (A^T A)^{-1} A^T \]

\[ = A^{-1} (A^T)^{-1} A^T = A^{-1} I \]

Put \( A' = A^{-1} I \) in all option.

- option (A) \[ AA'A = A \]
  \[ AA^{-1}A = A \]
  \[ A = A \] (true)

- option (B) \[ (AA')^2 = I \]
  \[ (AA^{-1}I)^2 = I \]
  \[ (I)^2 = I \] (true)

- option (C) \[ A'A = I \]
  \[ A^{-1}IA = I \]
  \[ I = I \] (true)

- option (D) \[ AA'A = A' \]
  \[ AA^{-1}A = A \neq A' \] (false)

**SOL 1.59**

Correct option is (A).

Given:
\[ M = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{2}{5} & \frac{3}{5} \end{bmatrix} \]

And \[ [M]^T = [M]^{-1} \]

We know that when \( [A]^T = [A]^{-1} \) then it is called orthogonal matrix.

\[ [M]^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

Substitute the values of \( M \) and \( M^T \), we get
\[
\begin{bmatrix}
\frac{3}{5} & \frac{4}{5} \\
\frac{2}{5} & \frac{3}{5}
\end{bmatrix}
\begin{bmatrix}
\frac{3}{5} & \frac{4}{5} \\
\frac{2}{5} & \frac{3}{5}
\end{bmatrix}
= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

\[
\begin{bmatrix}
\left( \frac{3}{5} \times \frac{3}{5} \right) + x^2 \\
\left( \frac{4}{5} \times \frac{3}{5} \right) + \frac{3}{5} \times \frac{3}{5}
\end{bmatrix}
= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]
Comparing both sides $a_{12}$ element,

$$\frac{12}{25} + \frac{3}{5}x = 0 \rightarrow x = -\frac{12}{25} \times \frac{5}{3} = -\frac{4}{5}$$

\[\begin{align*}
\text{SOL 1.60} & \\
\text{Correct answer is 0.5.} & \\
\begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
or & \\
\begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\end{align*}\]

So, $2x = 1 \Rightarrow x = \frac{1}{2}$

\[\begin{align*}
\text{SOL 1.61} & \\
\text{Correct option is (B).} & \\
\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} (2) (2) + (-1)(1) \\ (3)(2) + (2)(1) \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}
\end{align*}\]

\[\begin{align*}
\text{SOL 1.62} & \\
\text{Correct option is (C).} & \\
\begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}^T \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} (1)(1) + (2)(2) + (0)(0) & (1)(3) + (2)(-1) + (0)(4) \\ (3)(1) + (-1)(2) + 4(0) & (3)(3) + (-1)(-1) + (4)(4) \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 1 & 26 \end{bmatrix}
\end{align*}\]

\[\begin{align*}
\text{SOL 1.63} & \\
\text{Correct option is (B).} & \\
\begin{bmatrix} -1 & 7 \\ -1 & 20 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 7 \\ -1 & 20 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 7 \\ -1 & 20 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 7 \\ -1 & 20 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 20 \\ 1 & 7 \end{bmatrix}^{-1} = \begin{bmatrix} 26 & -13 \\ -13 & 65 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}
\end{align*}\]

\[\begin{align*}
\text{SOL 1.64} & \\
\text{Correct answer is 0.35.} & \\
\text{We have} & \\
\begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 0.5 & a \\ 0 & 1 \end{bmatrix}
\end{align*}\]

\[\begin{align*}
\text{Now} & \\
\begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 0.5 & a \\ 0 & 1 \end{bmatrix}
\end{align*}\]
or \[
\begin{bmatrix}
2 - 0.1 & \frac{1}{2} a \\
0 & 3
\end{bmatrix}
\begin{bmatrix}
b \\
o
\end{bmatrix}
= \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

or \[
\begin{bmatrix}
1 & 2a - 0.1b \\
0 & 3b
\end{bmatrix}
= \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

or \[2a - 0.1 = 0 \text{ and } 3b = 1\]

Thus solving above we have \[b = \frac{1}{3}\] and \[a = \frac{1}{60}\]

Therefore \[a + b = \frac{1}{3} + \frac{1}{60} = \frac{7}{20}\]

**SOL 1.65**

Correct option is (A).

We know that \[AA^{-1} = I\]

or \[
\begin{bmatrix}
2 - 0.1 & \frac{1}{2} a \\
0 & 3
\end{bmatrix}
\begin{bmatrix}
b \\
o
\end{bmatrix}
= \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

We get \[3b = 1 \text{ or } b = \frac{1}{3}\]

and \[2a - 0.1b = 0 \text{ or } a = \frac{b}{20}\]

Thus \[a + b = \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{20} = \frac{7}{20}\]

**SOL 1.66**

Correct option is (A).

We have \[\begin{bmatrix}
2 & 6 \\
3 & 9
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}\]

We get \[\begin{bmatrix}
6 + 6y & 2x + 12 \\
9 + 9y & 3x + 18
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}\]

We get \[6 + 6y = 0 \text{ or } y = -1\]

and \[2x + 12 = 0 \text{ or } x = -6\]

**SOL 1.67**

Correct option is (C).

\[
AB = \begin{bmatrix}
1 & 1 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix}
= \begin{bmatrix}
1(1) + 1(0) + 0(1) \\
1(1) + 0(0) + 1(1)
\end{bmatrix} = \begin{bmatrix}
1 \\
2
\end{bmatrix}
\]

**SOL 1.68**

Correct option is (A).

We have \[AB = \begin{bmatrix}
1 & 2 \\
2 & 0
\end{bmatrix}
\begin{bmatrix}
1 & 2 \\
0 & 1
\end{bmatrix} = \begin{bmatrix}
1 & 4 \\
4 & 4
\end{bmatrix}\]

\[(AB)^T = \begin{bmatrix}
1 & 4 \\
4 & 4
\end{bmatrix}\]
Correct option is (C).

\[ \mathbf{AB} = \begin{bmatrix} 2 & -1 & 1 & -2 & -5 \\ 1 & 0 & 3 & 4 & 0 \\ -3 & 4 & \end{bmatrix} \]

\[ = (2)(1) + (-1)(3) (2)(-2) + (-1)(4) (2)(-5) + (-1)(0) \]
\[ = (1)(1) + (0)(3) (1)(-2) + (0)(4) (1)(-5) + (0)(0) \]
\[ = (-3)(1) + (4)(3) (-3)(-2) + (4)(4) (-3)(-5) + (4)(0) \]
\[ = \begin{bmatrix} 2 & -1 & -8 & -10 \\ 1 & -2 & -5 & 1 \\ 9 & 22 & 15 \end{bmatrix} \]

Correct answer is 3.

We have
\[ \mathbf{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]
and transpose of \( \mathbf{X} \),
\[ \mathbf{X}^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \]

Here, we can see that rank of matrix \( \mathbf{X} = 3 \), hence, we can determine the rank of \( \mathbf{X}^T \mathbf{X} \).

Let \( \mathbf{Y} = \mathbf{X} \cdot \mathbf{X}^T \), the rank of \( \mathbf{Y} \leq \text{rank of } \mathbf{X} \). Also, \( \mathbf{X}^{-1} = \mathbf{X}^T \) and so we have

\[ \text{Rank } \mathbf{X} = \text{Rank } \mathbf{X}^T \leq \text{Rank of } \mathbf{Y} \]

Hence, from Eqs. (1) and (2), we get

\[ \text{Rank of } \mathbf{X} = \text{Rank of } \mathbf{Y} \]

Hence, rank of \( \mathbf{X} \cdot \mathbf{X}^T \) is 3

Correct option is (A).

\[ \mathbf{X}_{4 \times 3} \rightarrow \mathbf{X}^T_{3 \times 4} \]
\[ \mathbf{X}^T_{3 \times 4} \mathbf{Y}_{4 \times 3} \rightarrow (\mathbf{X}^T \mathbf{Y})_{3 \times 3} \]
\[ (\mathbf{X}^T \mathbf{Y})_{3 \times 3} \rightarrow (\mathbf{X}^T \mathbf{Y})^{-1}_{3 \times 3} \]
\[ \mathbf{P}_{2 \times 3} \rightarrow \mathbf{P}^T_{3 \times 2} \]
\[ (\mathbf{X}^T \mathbf{Y})_{3 \times 3} \mathbf{P}_{3 \times 2} \rightarrow (\mathbf{X}^T \mathbf{Y})_{3 \times 3} \mathbf{P}^T_{3 \times 2} \]
\[ \mathbf{P}_{2 \times 3} (\mathbf{X}^T \mathbf{Y})_{3 \times 3} \mathbf{P}^T_{3 \times 2} \rightarrow (\mathbf{X}^T \mathbf{Y})_{3 \times 3} \mathbf{P}^T_{3 \times 2} \]
\[ \mathbf{P}_{2 \times 3} (\mathbf{X}^T \mathbf{Y})_{3 \times 3} \mathbf{P}_{3 \times 2} \rightarrow (\mathbf{X}^T \mathbf{Y})_{3 \times 3} \mathbf{P}_{3 \times 2} \]

Correct option is (C).

\[ \mathbf{A}_\alpha \cdot \mathbf{A}_\beta = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix} \]
\[ = \begin{bmatrix} \cos(\alpha + \beta) & \sin(\alpha + \beta) \\ -\sin(\alpha + \beta) & \cos(\alpha + \beta) \end{bmatrix} = \mathbf{A}_{\alpha + \beta} \]
Also, it is easy to prove by induction that
\[(A_n)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}\]

**SOL 1.73**
Correct option is (D).
Let \(\tan \frac{\alpha}{2} = t\).

Then,
\[
\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} = \frac{1 - t^2}{1 + t^2}
\]
and
\[
\sin \alpha = \frac{2\tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} = \frac{2t}{1 + t^2}
\]

\[
(I - A)[\begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}] = \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix}[\begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}] = \begin{bmatrix} \frac{1 - t^2}{1 + t^2} & \frac{2t}{1 + t^2} \\ \frac{-2t}{1 + t^2} & \frac{1 - t^2}{1 + t^2} \end{bmatrix}
\]

\[= \begin{bmatrix} 1 + t & -t \\ -t & 1 \end{bmatrix} \begin{bmatrix} 1 - \tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} \end{bmatrix} = (I + A)
\]

**SOL 1.74**
Correct option is (C).
\[
AAT = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & -\cos \theta \sin \theta + \sin \theta \cos \theta \\ -\sin \theta \cos \theta + \cos \theta \sin \theta & \sin^2 \theta + \cos^2 \theta \end{bmatrix}
\]
\[= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

Hence \(A\) is an orthogonal matrix.

**SOL 1.75**
Correct option is (C).
\[
AB = \begin{bmatrix} \cos \theta \cos \phi \cos(\theta - \phi) & \cos \alpha \sin \phi \cos(\theta - \phi) \\ \cos \phi \sin \alpha \cos(\alpha - \phi) & \sin \theta \sin \phi \cos(\theta - \phi) \end{bmatrix}
\]

Is a null matrix when \(\cos(\theta - \phi) = 0\), this happens when \((\theta - \phi)\) is an odd multiple of \(\frac{\pi}{2}\).

**SOL 1.76**
Correct option is (C).
Let matrix \(A\) be \(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\).

From \(A \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}\) we get \(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} a - b \\ c - d \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}\)

We have \(a - b = -1\) \(\ldots(1)\)
and \(c - d = 1\) \(\ldots(2)\)
From $\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix} = -2 \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$ we get

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} a - 2b \\ c - 2d \end{bmatrix} = -2 \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

we have

$$a - 2b = -2$$

and

$$c - 2d = 4$$

...(2)

Solving equation (1) and (3) $a = 0$ and $b = 1$

Solving equation (2) and (4) $c = -2$ and $d = -3$

Thus

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

If we check all option then result of option C after multiplication gives result.

**SOL 1.77**

Correct option is (D).

$$A^2 = \begin{bmatrix} 3 & -4 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$$

If we put $n = 2$ in option, then only D satisfy.

**SOL 1.78**

Correct option is (A).

$$\begin{bmatrix} 3 & 1 & 2 & -1 \\ b & k_1 & 3 + k_2 & 6 + k_3 & -3 \\ c & 2 & 4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 3 + 2k_2 - k_3 \end{bmatrix} = \begin{bmatrix} 3 \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ b \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ c \end{bmatrix}$$

$$k_1 + 2k_2 - k_3 = 3$$

$$3k_1 + 6k_2 - 3k_3 = b$$

$$2k_1 + 6k_2 - 2k_3 = c$$

$$\Rightarrow b = 9,$$

$$c = 6$$

**SOL 1.79**

Correct option is (B).

$$\det(A) = (ah - cf)k + bef + cdg - aeg - bdh$$

Thus matrix $A$ is invertible for all $k$ if (and only if) the coefficient $(ah - cf)$ of $k$ is 0, while the sum $bef + cdg - aeg - bdh$ is non zero.

$$\Rightarrow$$ Thus infinitely many other soln

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