Q.1– Q.5 Carry One Mark Each.

Q1 Which of the following is CORRECT with respect to grammar and usage?
Mount Everest is _____
(A) the highest peak in the world
(B) highest peak in the world
(C) one of highest peak in the world
(D) One of the highest peak in the world

S1 Correct option is (A).
Before superlative article ‘the’ has to be used. ‘one of’ the expression should take plural noun and so option ‘C’ and ‘D’ can’t be the answer.

Q2 The policeman asked the victim of a theft, “What did you _____ ?”
(A) loose (B) lose
(C) loss (D) louse

S1 Correct option is (B).
‘lose’ is verb.

Q3 Despite the new medicine’s _____ in treating diabetes, it is not _____ widely.
(A) Effectiveness ______ prescribed
(B) availability ______ used
(C) prescription ______ available
(D) acceptable ______ proscribed

S1 Correct option is (A).
‘effectiveness’ is noun and ‘prescribed’ is verb. These words are apt and befitting with the word ‘medicine.’

Q4 In a huge pile of apples and oranges, both ripe and unripe mixed together, 15% are unripe fruits, of the unripe fruits, 45% are apples, of the ripe ones, 66% are oranges. If the pile contains a total of 5692000 fruits, how many of them are apples?
(A) 2029198 (B) 2467482
(C) 2789080 (D) 3577422

S1 Correct option is (A).
Total no. of fruits = 5692000
Unripe type of apples = 45% of 15% of 5692000
= \( \frac{45}{100} \times \frac{15}{100} \times 5692000 \)
= 384210
Ripe type of apples = \( \frac{34}{100} \times \frac{85}{100} \times 5692000 \)
= 1644988
.: Total no. of apples = 384210 + 1644988 = 2029198

Q5 Michael lives 10 km way from where I live. Ahmed lives 5 km away and Susan lives 7 km away from where I live. Arun is farther away than Ahmed but closer than Susan from where I live. From the information provided here, what is one possible distance (in km) at which I live from Arun’s place?
(A) 3.00 (B) 4.99
(C) 6.02 (D) 7.01

S1 Correct option is (C).
From given data, the following diagram is possible.

\[ \text{I} \quad \text{AH} \quad \text{M} \]
\[ \text{S} \quad 7 \text{ km} \]
\[ \text{5 km} \]
\[ \text{10 km} \]

I = I live
AH = Ahmed lives
M = Michael lives
S = Susan lives
A = Arun lives

Arun lives farther away than Ahmed means more than 5 km but closer than Susan means less than 7 km, from given alternatives, option (C) only possible.

Q.6 – Q.10 Carry two marks each.

Q6 A person moving through a tuberculosis prone zone has a 50% probability of becoming infected. However, only 30% of infected people develop the disease. What percentage of people moving through a tuberculosis prone zone remains infected but does ot shows symptoms of disease?
(A) 15 (B) 33
(C) 35 (D) 37

S1 Correct option is (C).

\[ \text{Tuberculosis} \]
\[ \text{Affected} \quad 50\% \]
\[ \text{Infected} \quad 50\% \]
\[ \text{Develop the disease} \quad 30\% \text{ of } 50\% \]
\[ \text{Does not} \quad \text{Develop the disease} \quad 70\% \text{ of } 50\% \]
\[
= \frac{70}{100} \times \frac{50}{100} = \frac{35}{100} = 35\%
\]

Q7  In a world filled with uncertainty, he was glad to have many good friends. He had always assisted them in times of need and was confident that they would reciprocate. However, the events of the last week proved him wrong. Which of the following inference(s) is/are logically valid and can be inferred from the above passage?
(i) His friends were always asking him to help them.
(ii) He felt that when in need of help, his friends would let him down.
(iii) He was sure that his friends would help him when in need.
(iv) His friends did not help him last week.
(A) (i) and (ii)  (B) (iii) and (iv)  (C) (iii) only  (D) (iv) only

S1  Correct option is (B).
The words ‘was confident that they would reciprocate’ and ‘last week proved him wrong’ lead to statements (iii) and (iv) as logically valid inferences.

Q8  Leela is older than her cousin Pavithra, Pavithra’s brother Shiva is older than Leela. When Pavithra and Shiva are visiting Leela, all three like to play chess. Pavithra wins more often than Leela does. Which one of the following statements must be TRUE based on the above?
(A) When Shiva plays chess with Leela and Pavithra, He often loses.
(B) Leela is the oldest of the three.
(C) Shiva is a better chess player than Pavithra.
(D) Pavithra is the youngest of the three.

S1  Correct option is (D).
From given data, the following arrangement is possible.
Shiva
Leela
Pavithra
Among four alternatives, option (D) is TRUE.

Q9  If \( q^a = \frac{1}{r} \) and \( r^b = \frac{1}{s} \) and \( s^c = \frac{1}{q} \), the value of \( abc \) is
(A) (rqs)-1
(B) 0
(C) 1
(D) \( r + q + s \)

S1  Correct option is (C).
\[
q^a = \frac{1}{r} \Rightarrow \frac{1}{q^a} = r \\
r^b = \frac{1}{s} \Rightarrow \frac{1}{r^b} = s \\
s^c = \frac{1}{q} \Rightarrow \frac{1}{s^c} = q \\
q^a = r \Rightarrow (s^c)^a = r \Rightarrow s^{ac} = r
\]

Q10  \( P, Q, R \) and \( S \) are working on a project. \( Q \) can finish the task in 25 days, working alone for 12 hours a day. \( R \) can finish the task in 50 days, working alone for 12 hours per day. \( Q \) worked 12 hours a day but took sick leave in the beginning for two days. \( R \) worked 18 hours a day on all days. What is the ratio of work done by \( Q \) and \( R \) after 7 days from the start of the project.
(A) 10 : 11  (B) 11 : 10  (C) 20 : 21  (D) 21 : 20

S1  Correct option is (C).
\[
Q \text{ working hours } = 12 \times (7 - 2) = 60 \text{ hrs} \\
R \text{ working hours } = 18 \times 7 = 126 \text{ hrs}
\]
After 7 days, the ratio of work done by \( Q \) and \( R \)
\[
Q : R = \frac{60}{126} = \frac{20}{42} = \frac{20}{50} = 20 : 21
\]

Q.1 – Q.25 Carry One Mark Each.

Q11  Let \( M^I = I \) (where \( I \) denotes the identity matrix) and \( M \neq I, M^2 \neq I \) and \( M^3 \neq I \). Then, for any natural number \( k \), \( M^k \) equals:
(A) \( M^{4k+1} \)  (B) \( M^{4k+2} \)
(C) \( M^{4k+3} \)  (D) \( M^{4k+4} \)

S1  Correct option is (C).
\[
\Rightarrow M^6 = M^4 = I \Rightarrow M^2 = M^4 \Rightarrow M^2 = M^1 \\
\Rightarrow M^4 = M^2 = I \Rightarrow M^4 = M^1 \Rightarrow M^4 = M^6 = M^1
\]
\[\therefore M^{-1} = M^{4k+3}, K \text{ is a natural number.}\]

Q12  The second moment of a Poisson-distributed random variable is 2. The mean of the random variable is ________.

S1  Correct answer is \( \lambda = 1 \).
\[
E(x^2) = 2 \\
V(X) = E(X^2) - (E(X))^2 \\
\text{ Let mean of the poission random variable be } x \\
x = 2 - x^2 \\
x^2 + x - 2 = 0 \\
x = 1, -2
\]
\[\therefore \text{ Means is } \lambda = 1\]
Q13 Given the following statements about a function
\( f : \mathbb{R} \rightarrow \mathbb{R} \), select the right option:
P : If \( f(x) \) is continuous at \( x = x_0 \), then it is also
differentiable at \( x = x_0 \)
Q : If \( f(x) \) is continuous at \( x = x_0 \), then it may not be
differentiable at \( x = x_0 \)
R : If \( f(x) \) is differentiable at \( x = x_0 \), then it is also
continuous at \( x = x_0 \)
(A) P is true, Q is false, R is false
(B) P is false, Q is true, R is true
(C) P is false, Q is true, R is false
(D) P is true, Q is false, R is true

S1 Correct option is (B).
Since continuous function may not be differentiable. But
differentiable function is always continuous.

Q14 Which one of the following is a property of the
solutions to the Laplace equation : \( \nabla^2 f = 0 \)?
(A) The solutions have neither maxima nor minima any-
where except at the boundaries
(B) The solutions are not separable in the coordinates
(C) The solutions are not continuous
(D) The solutions are not dependent on the boundary
conditions

S1 Correct option is (A).

Q15 Consider the plot of \( f(x) \) versus \( x \) as shown below.
Suppose \( F(x) = \int_{-5}^{x} f(y) \, dy \). Which one of the following is
a graph of \( F(x) \)?

S1 Correct option is (C).
Integration of ramp is parabolic, integration of step is ramp.

Q16 Which one of the following is an eigen function of
the class of all continuous-time, linear, time-invariant
systems (\( u(t) \) denotes the unit-step function)?
(A) \( e^{j\omega t} u(t) \)
(B) \( \cos(\omega t) \)
(C) \( e^{-j\omega t} \)
(D) \( \sin(\omega t) \)

S1 Correct option is (C).
If the input to a system is its eigen signal, the response
has the same form as the eigen signal.

Q17 A continuous-time function \( x(t) \) is periodic with
period, \( T \). The function is sampled uniformly with a
sampling period \( T_s \). In which one of the following cases is
the sampled signal periodic?
(A) \( T = \sqrt{2} T_s \)
(B) \( T = 1.2 T_s \)
(C) Always
(D) Never

S1 Correct answer is (B).
A discrete time signal \( x(n) = \cos(\omega_n n) \) is said to be
periodic if \( \omega_n = \frac{2\pi}{2\pi} \) is a rational number

Q18 Consider the sequence \( x[n] = a^n u[n] + b^n u[n] \), where
\( u[n] \) denotes the unit-step sequence and \( 0 < |a| < |b| < 1 \).
The region of convergence (ROC) of the z-transform of
\( x[n] \) is
(A) \( |z| > |a| \)
(B) \( |z| > |b| \)
(C) \( |z| < |a| \)
(D) \( |a| < |z| < |b| \)
**S1** Correct option is (B).
\[ x(n) = (a)^n x(n) + (b)^n x(n), \text{ given } 0 < |a| < |b| < 1 \]
\[ R_{oc} = \left| z \right| > |a| \cap \left| z \right| > |b| \]

**Q19** Consider a two-port network with the transmission matrix: \( T = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \). If the network is reciprocal, then
(A) \( T^{-1} = T \)
(B) \( T^T = T \)
(C) Determinant \( (T) = 0 \)
(D) Determinant \( (T) = 1 \)

**S1** Correct option is (D).
A two port network is reciprocal in transmission parameters if \( AD - BC = 1 \) i.e. Determinant \( (T) = 1 \)

**Q20** A continuous-time sinusoid of frequency 33 Hz is multiplied with a periodic Dirac impulse train of frequency 46 Hz. The resulting signal is passed through an ideal analog lowpass filter with a cutoff frequency of 23 Hz. The functional frequency (in Hz) of the output is __________.

**S1** Correct answer is 13.

\( f_s = 33 \text{ Hz, } f_L = 46 \text{ Hz} \)
The frequency in sampled signal are = 33, 13, 79, 59, 125, ....
The above frequencies are passed to a LPF of cutoff frequency 23 Hz. The output frequency is = 13 Hz.

**Q21** A small percentage of impurity is added to intrinsic semiconductor at 300 K. Which one of the following statements is true for the energy band diagram shown in the following figure?

<table>
<thead>
<tr>
<th>Conduction Band</th>
<th>( E_C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Energy Level</td>
<td>0.01 \text{ eV}</td>
</tr>
<tr>
<td>Valence Band</td>
<td>( E_V )</td>
</tr>
</tbody>
</table>

(A) Intrinsic semiconductor doped with pentavalent atoms to form \( n \)-type semiconductor
(B) Intrinsic semiconductor doped with trivalent atoms to form \( n \)-type semiconductor
(C) Intrinsic semiconductor doped with pentavalent atoms to form \( p \)-type semiconductor
(D) Intrinsic semiconductor doped with trivalent atoms to form \( p \)-type semiconductor

**S1** Correct option is (A).
Donor energy level close to conduction band.

**Q22** Consider the following statements for a metal oxide semiconductor field effect transistor (MOSFET):
P: As channel length reduces, OFF-state current increases
Q: As channel length reduces, output resistance increases
R: As channel length reduces, threshold voltage remains constant
S: As channel length reduces, ON current increases.
Which of the above statements are INCORRECT?
(A) P and Q
(B) P and S
(C) Q and R
(D) R and S

**S1** Correct option is (C).
P: TRUE
Q: FALSE, As channel length reduces, output resistance reduces
R: FALSE: As channel length reduces, threshold voltage reduces
S: TRUE

**Q23** Consider the constant current source shown in the figure below. Let \( \beta \) represent the current gain of the transistor.

The load current \( I_0 \) through \( R_L \) is
(A) \( I_0 = \left( \frac{\beta + 1}{\beta} \right) \frac{V_{dd}}{R} \)
(B) \( I_0 = \left( \frac{\beta}{\beta + 1} \right) \frac{V_{dd}}{R} \)
(C) \( I_0 = \left( \frac{\beta + 1}{\beta} \right) \frac{V_{dd}}{2R} \)
(D) \( I_0 = \left( \frac{\beta}{\beta + 1} \right) \frac{V_{dd}}{2R} \)

**S1** Correct option is (B).

\[ VP = V_N \text{ (Virtual short)} \]
\[ I_0 = I_C = \left( \frac{\beta}{\beta + 1} \right) \frac{V_N}{R} \]

**Q24** The following signal \( V_i \) of peak voltage 8 V is applied to the non-inverting terminal of an ideal Opamp.
The transistor has $V_{BE} = 0.7 \text{ V}$; $\beta = 100$; $V_{LED} = 1.5 \text{ V}$, $V_{CC} = 10 \text{ V}$ and $-V_{CC} = -10 \text{ V}$.

The circuit shown is a wein bridge oscillator. The amplitude of oscillations can be determined and stabilized by using a nonlinear control network. As the oscillations grow, the diodes start to conduct causing the effective resistance in the feedback to decrease. Equilibrium will be reached at the output amplitude that causes the loop gain to be exactly unity.

**Q26** The block diagram of a frequency synthesizer consisting of Phase Locked Loop (PLL) and a divide-by-N counter (comprising $\div2$, $\div4$, $\div8$, $\div16$ outputs) is sketched below. The synthesizer is excited with a 5 kHz signal (Input 1). The free-running frequency of the PLL is set to 20 kHz. Assume that the commutator switch makes contacts repeatedly in the order 1-2-3-4.
The corresponding frequencies synthesized are:
(A) 10 kHz, 20 kHz, 40 kHz, 80 kHz
(B) 20 kHz, 40 kHz, 80 kHz, 160 kHz
(C) 80 kHz, 40 kHz, 20 kHz, 10 kHz
(D) 160 kHz, 80 kHz, 40 kHz, 20 kHz

Correct option is (A).

<table>
<thead>
<tr>
<th>$f_{in}$</th>
<th>Divide By N</th>
<th>VCO output ($Nf_{in}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 kHz</td>
<td>2</td>
<td>10 kHz</td>
</tr>
<tr>
<td>5 kHz</td>
<td>4</td>
<td>20 kHz</td>
</tr>
<tr>
<td>5 kHz</td>
<td>8</td>
<td>80 kHz</td>
</tr>
<tr>
<td>5 kHz</td>
<td>16</td>
<td>80 kHz</td>
</tr>
</tbody>
</table>

The output of the combinational circuit given below is

- (A) $A + B + C$
- (B) $A(B + C)$
- (C) $B(C + A)$
- (D) $C(A + B)$

Correct option is (C).

What is the voltage $V_{out}$ in the following circuit?

- (A) 0 V
- (B) $\frac{[V_t \text{ of PMOS}] + V_t \text{ of NMOS}}{2}$
- (C) Switching threshold of inverter
- (D) $V_{dd}$

Correct option is (C).

Match the inferences $X$, $Y$ and $Z$, about a system, to the corresponding properties of the elements of first column in Routh’s Table of the system characteristic equation.

- $X$: The system is stable ...
- $Y$: The system is unstable ...
- $Z$: The test breaks down ....
- $P$: ... When all elements are positive
- $Q$: .... When any one element is zero
- $R$: .... When there is a change in sign of coefficients

- (A) $XP$, $YQ$, $ZR$
- (B) $XQ$, $YP$, $ZR$
- (C) $XR$, $YQ$, $ZP$
- (D) $XP$, $YR$, $ZQ$

Correct option is (D).

A closed-loop control system is stable if the Nyquist plot of the corresponding open loop transfer function

- (A) encircles the $s$-plane point $(-1 + j0)$ in the counter-clockwise direction as many times as the number of right-half $s$-plane poles.
- (B) encircles the $s$-plane point $(0 - j1)$ in the clockwise direction as many times as the number of right-half $s$-plane poles.
(C) encircles the s-plane point \((-1 + \jmath 0)\) in the counter-clockwise direction as many times as the number of left-half s-plane poles.

(D) encircles the s-plane point \((-1 + \jmath 0)\) in the counter-clockwise direction as many times as the number of right-half s-plane zeros.

S1 Correct option is (A).

\[
N = P - Z
\]

For closed loop stability \(Z = 0\), \(N = P\)

\[\therefore (-1, \jmath 0) \text{ should be encircled in Counter clock wise direction equating } P \text{ poles in RHP.}\]

Q31 Consider binary data transmission at a rate of 56 kbps using baseband binary pulse amplitude modulation (PAM) that is designed to have a raised-cosine spectrum. The transmission bandwidth (in kHz) required for a roll-off factor of 0.25 is ________.

S1 Correct answer is 35.

\[R_s = 56 \text{ kbps}, \quad \alpha = 0.2\]

\[BW = \frac{R_s}{2}[1 + \alpha] = \frac{56}{2}[1 + 0.25]\text{kHz} = 35 \text{kHz}\]

Q32 A superheterodyne receiver operates in the frequency range of 58 MHz-68 MHz. The intermediate frequency \(f_{s1}\) and local oscillator frequency \(f_{lo}\) are chosen such that \(f_{s1} \leq f_{lo}\). It is required that the image frequencies fall outside the 58 MHz - 68 MHz band. The minimum required \(f_{s1}\) (in MHz) is ________.

S1 Correct answer is 5.

\[f_s = 58 \text{ MHz - 68 MHz}\]

When \(f_s = 58 \text{ MHz}\)

\[f_s = f_s + 21F > 68 \text{ MHz}\]

\[21F > 10 \text{ MHz}\]

\[1F \geq 5 \text{ MHz}\]

Q33 The amplitude of a sinusoidal carrier is modulated by a single sinusoid to obtain the amplitude modulated signal \(s(t) = 5\cos 1600\pi t + 20\cos 1800\pi t + 5\cos 2000\pi t\). The value of the modulation index is ________.

S1 Correct answer is \(\mu = 0.5\).

\[S(f) = 5\cos 1600\pi t + 20\cos 1800\pi t + 5\cos 2000\pi t\]

\[S(f) = \frac{A}{\mu} \cos 2\pi (f_s - f_c) t + A_s \cos 2\pi f_c t\]

\[\frac{A}{\mu} = 20, \quad A_s = 10\]

\[\frac{A}{\mu} = 5, \quad \mu = 0.5\]

Q34 Concentric spherical shells of radii 2 m, 4 m and 8 m carry uniform surface charge densities of 20 nC/m², -4 nC/m² and \(\rho_s\), respectively. The value of \(\rho_s\) (nC/m²) required to ensure that the electric flux density \(D = 0\) at radius 10 m is ________.

S1 Correct answer is -0.25.

\[\rho_s = 20 \text{ nC/m²}\]

\[\rho_s = -4 \text{ nC/m²}\]

\[\rho_s = ? \text{ (unknown)}\]

Electric flux density at \(r = 10 \text{ m}\) in given by

\[D = \frac{\text{Area of sphere of radius } r = 10 \text{ m}}{\pi r^2}\]

\[D = 0\]

\[\psi = \psi_n + \sum_{(\omega)} = 0;\]

\[20 \times 10^{-9} \times 4\pi (2)^2 + (-4 \times 10^{-9})\]

\[\times 4\pi (4)^2 + \rho_{s0} 4\pi (8)^2 = 0\]

\[\rho_{s0} = \rho_s = -0.25 \text{ nc/m²}\]

Q35 The propagation constant of a lossy transmission line is \((2 + \jmath 0) \mu^{-1}\) and its characteristic impedance is \((50 + \jmath 0) \Omega\) at \(\omega = 10^6 \text{ rad/s}^{-1}\). The values of the line constants \(L, C, R, G\) are, respectively,

(A) \(L = 200 \mu \text{H/m},\; C = 0.1 \mu \text{F/m},\; R = 50 \Omega/m,\; G = 0.02 \text{ S/m}\)

(B) \(L = 250 \mu \text{H/m},\; C = 0.1 \mu \text{F/m},\; R = 100 \Omega/m,\; G = 0.04 \text{ S/m}\)

(C) \(L = 200 \mu \text{H/m},\; C = 0.2 \mu \text{F/m},\; R = 100 \Omega/m,\; G = 0.02 \text{ S/m}\)

(D) \(L = 250 \mu \text{H/m},\; C = 0.2 \mu \text{F/m},\; R = 50 \Omega/m,\; G = 0.04 \text{ S/m}\)

S1 Correct option is (B).

Propagation contact, \(P = (2 + \jmath 0) \mu^{-1}\), characteristic impedance \(Z_0 = 50 \Omega\), angular frequency \(\omega = 10^6 \text{ rad/sec}\),

\[P = \sqrt{(R + \jmath \omega L)(G + \jmath \omega C)}\]

\[Z_0 = \sqrt{(R + \jmath \omega L)(G + \jmath \omega C)}\]

\[P_{\text{in}} = R + \jmath \omega L\]

\[R = 100 \Omega/m\]

\[L = \frac{250}{10^6} = 250 \mu \text{H/m}\]
\[
\frac{P}{\omega_0} = G + j\omega C \\
G + j\omega C = \left( \frac{2}{30} + j\frac{5}{60} \right) \\
\therefore \quad G = 0.04 \text{ s/m} \\
C = \frac{5}{50 \times 10^{-9}} = 0.2 \mu F/\text{m}
\]

Therefore line constants \( L, C, R \& G \) are respectively 
\( L = 250 \mu \text{H/m}, \quad C = 0.1 \mu \text{F/m}, \quad R = 100 \Omega/\text{m}, \quad G = 0.04 \text{ s/m} \)

Q.26 – Q.55 Carry Two Marks Each.

**Q36** The integral \( \frac{1}{2\pi} \int \int_D (x + y + 10) \, dx \, dy \), where \( D \) denotes the disc: \( x^2 + y^2 \leq 4 \), evaluates to _____.

**S1** Correct answer is 20.

Converting to polar coordinates, we get
\[
\frac{1}{2\pi} \int \int_D (x + y + 10) \, dx \, dy = \frac{1}{2\pi} \int_{\theta=0}^{2\pi} \int_{r=0}^{2} (r^2 \cos \theta + r^2 \sin \theta + 10r) \, r \, dr \, d\theta = 20
\]

**Q37** A sequence \( x[n] \) is specified as
\[
\begin{bmatrix}
  x[n] \\
  x[n-1]
\end{bmatrix} = \begin{bmatrix}
  1 & 1 & 0 \\
  1 & 0 & 1
\end{bmatrix} \quad \text{for } n \geq 2
\]
The initial conditions are \( x[0] = 1, x[1] = 1 \) and \( x[n] = 0 \) for \( n < 0 \). The value of \( x[12] \) is _____

**S1** Correct answer is 233.

\[\begin{array}{c}
  x(n) \\
  x(n-1)
\end{array} = \begin{array}{c}
  1 & 1 & 0 \\
  1 & 0 & 1
\end{array} \quad \text{for } n \geq 2
\]
\[
\begin{array}{c}
  x(2) \\
  x(1)
\end{array} = \begin{array}{c}
  2 & 2 & 1 \\
  1 & 1 & 1
\end{array} \quad \text{for } x(2) = 2, x(1) = 1
\]
\[
\begin{array}{c}
  x(3) \\
  x(2)
\end{array} = \begin{array}{c}
  3 & 2 & 2 \\
  1 & 2 & 1
\end{array} \quad \text{for } x(3) = 3, x(2) = 2
\]

From the above values we can write the recursive relation as
\( x(n) = x(n-1) + x(n-2) \)

**Q38** In the following integral, the contour \( C \) encloses the points \( 2\pi j \) and \( -2\pi j \).

The value of the integral \( \frac{-1}{2\pi} \int_C \sin z (z - 2\pi j)^3 \, dz \) is _____.

**S1** Correct answer is –133.8.

\[
\frac{-1}{2\pi} \int_C \sin z (z - 2\pi j)^3 \, dz = \frac{-1}{2\pi} \times 2\pi j \int_{-\pi}^{\pi} \sin (2\pi j) \, \frac{\sinh (2\pi j)}{2} \]
\[
\therefore \quad f''(z) = -\sin z
\]
\[
\frac{-1}{2\pi} \int_C \sin z (z - 2\pi j)^3 \, dz = -\frac{1}{2\pi} \times 2\pi j \left( -\frac{\sin (2\pi j)}{2} \right) \]
\[
= \frac{j}{2} \sinh 2\pi
\]
\[
= \frac{1}{2} \left( \sinh 2\pi \right)
\]
\[
= -133.87
\]

**Q39** The region specified by \( \{(\rho, \phi, Z) : 3 \leq \rho \leq 5, \frac{\pi}{8} \leq \phi \leq \frac{\pi}{4}, 3 \leq z \leq 4.5\} \) in cylindrical coordinates has volume of _____.

**S1** Correct answer is 4.714.

Given region of cylinder
\( 3 \leq \rho \leq 5, \frac{\pi}{8} \leq \phi \leq \frac{\pi}{4}, 3 \leq z \leq 4.5 \)

The differential volume of cylinder in given by
\[
dv = \rho d\rho d\phi dz
\]

Volume,
\[
v = \int_{\rho=3}^{5} \int_{\phi=\frac{\pi}{8}}^{\frac{\pi}{4}} \int_{z=3}^{4.5} \rho d\rho d\phi dz
\]
\[
= \frac{4}{3} \times \phi \frac{\pi}{4} \times z \frac{25}{2}
\]
\[
= \frac{1}{2} (25 - 9) \times \left( \frac{\pi}{4} - \frac{\pi}{8} \right) \times (4.5 - 3)
\]

**Q40** The Laplace transform of the causal periodic square wave of period \( T \) shown in the figure below is
(A) \( F(s) = \frac{1}{1 + e^{-sT}} \)  
(B) \( F(s) = \frac{1}{s(1 + e^{-sT})} \)  
(C) \( F(s) = \frac{1}{s(1 + e^{-sT})} \)  
(D) \( F(s) = \frac{1}{1 - e^{-sT}} \)  

S1 Correct option is (B).

One period of signal

\[
x_i(t) = u(t) - u\left(t - \frac{T}{2}\right)
\]

\[
X_i(s) = \frac{1}{s} \left[ \frac{e^{-sT/2}}{s} - 1 + e^{-sT/2} \right]
\]

\[
X(s) = \frac{1}{1 - e^{-sT}} X_i(s) = \frac{1}{s(1 - e^{-sT})}
\]

\[
= \frac{1}{s(1 + e^{-sT})}
\]

Q41 A network consisting of a finite number of linear resistor (R), inductor (L), and capacitor (C) elements, connected all in series or all in parallel, is excited with a source of the form \( \sum a_k \cos(k\omega_0 t) \), where \( a_k \neq 0 \), \( \omega_0 \neq 0 \). The source has nonzero impedance. Which one of the following is a possible form of the output measured across a resistor in the network?

(A) \( \sum b_k \cos(k\omega_0 t + \phi_k) \), where \( b_k \neq a_k \), \( \forall k \)

(B) \( \sum b_k \cos(k\omega_0 t + \phi_k) \), where \( b_k \neq 0 \), \( \forall k \)

(C) \( \sum a_k \cos(k\omega_0 t + \phi_k) \)

(D) \( \sum a_k \cos(k\omega_0 t + \phi_k) \)

S1 Correct option is (A).

S1 Correct option is (C).

\[
H(s) = \frac{1}{s + \frac{T}{2}}
\]

\[
V_0(s) = H(s) \cdot V_i(s)
\]

(I) if \( V_i(t) = \delta(t) \)

\[
V_i(s) = \frac{1}{s}
\]

\[
V_0(s) = H(s) \cdot V_i(s)
\]

\[= \frac{1}{s + \frac{T}{2}}
\]

\[
V_0(t) = \frac{e^t}{t}
\]

(II) If \( V_i(t) = u(t) \)

\[
V_i(s) = \frac{1}{s}
\]

\[
V_0(s) = \frac{1}{s + \frac{T}{2}} \cdot \frac{1}{s + \frac{1}{T}}
\]

\[
V_0(t) = (1 - e^{-\tau})
\]

(III) \( V_i(t) = r(t) \)

\[
V_i(s) = \frac{1}{s^3}
\]

\[
V_0(s) = H(s) \cdot V_i(s)
\]

\[= \frac{1}{s^3(1 + \frac{T}{2})}
\]

\[
= \frac{1}{s^3} - \frac{T}{s} + \frac{T}{s + \frac{1}{T}}
\]

\[
V_0(t) = t - T(1 - e^{-\tau})
\]

Q43 An AC voltage source \( V = 10 \sin(t) \) volts is applied to the following network. Assume that \( R_1 = 3 \, \Omega \), \( R_2 = 6 \, \Omega \) and \( R_3 = 9 \, \Omega \), and that the diode is ideal.

Consider a series RLC-Circuit with voltage source

Here

\[
V(t) = a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + a_3 \cos 3\omega_0 t
\]

\[
i(t) = b_1 \cos (\omega_0 t + \phi_1) + b_2 \cos (2\omega_0 t + \phi_2) + b_3 \cos (3\omega_0 t + \phi_3)
\]

Where \( b_k \neq a_k \) for all \( k \)

Q42 A first-order low-pass filter of time constant \( T \) is excited with different input signals (with zero initial conditions up to \( t = 0 \)). Match the excitation signals \( X \), \( Y \), \( Z \) with the corresponding time responses for \( t \geq 0 \):

- \( X \): Impulse \( \rightarrow P = 1 - e^{-t/T} \)
- \( Y \): Unit step \( \rightarrow Q = t - T(1 - e^{-t/T}) \)
- \( Z \): Ramp \( \rightarrow R = e^{-t/T} \)

(A) \( X \rightarrow R, Y \rightarrow Q, Z \rightarrow P \)
We can join nodes that are at same potential so network becomes

\[
P_{\text{max}} = \frac{V^2}{4R_L} \text{ W}
\]

\[
= \frac{(160)^2}{4 \times 8 \text{k}} = 0.8 \text{ W}
\]

**Q45** Consider the signal \( x[n] = 6\delta[n+2] + 3\delta[n+1] + 8\delta[n] + 7\delta[n-1] + 4\delta[n-2] \). If \( X(e^{j\omega}) \) is the discrete-time Fourier transform of \( x[n] \), then \( \frac{1}{\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\sin^2(2\omega) \, d\omega \) is equal to ________.

**S1** Correct answer is 0.8.

Plancheiral’s relation is

\[
\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \cdot Y(e^{j\omega}) \, d\omega = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n)
\]

\[
Y(e^{j\omega}) = \sin^2(2\omega)
\]

\[
= \frac{1 - \cos(4\omega)}{2}
\]

\[
y(n) = \frac{1}{2} \delta(n) - \frac{1}{2} \delta(n+4) - \frac{1}{4} \delta(n-4)
\]

\[
y(n) = \left\{ -\frac{1}{4}, 0, 0, 0, \frac{1}{4}, 0, 0, 0, -\frac{1}{4} \right\}
\]

\[
x(n) = \{6, 3, 8, 7, 4\}
\]

\[
\frac{1}{\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \cdot Y(e^{j\omega}) \, d\omega = 2 \sum_{n=-\infty}^{\infty} x(n) \cdot y(n)
\]

\[
= 2 \sum_{n=\infty}^{\infty} x(n) \cdot y(n)
\]

\[
= 2 \times 8 \times \frac{1}{2} = 8
\]

**Q46** Consider a silicon p-n junction with a uniform acceptor doping concentration of \( 10^{17} \text{ cm}^{-3} \) on the p-side and a uniform donor doping concentration of \( 10^{16} \text{ cm}^{-3} \) on the n-side. No external voltage is applied to the diode. Given: \( kT/q = 26 \text{ mV} \), \( n_i = 1.5 \times 10^{10} \text{ cm}^{-3} \), \( \epsilon_0 = 12\epsilon_0 \), \( \epsilon_0 = 8.85 \times 10^{-14} \text{ F/m} \), and \( q = 1.6 \times 10^{-19} \text{ C} \). The charge per unit junction area (nC cm\(^{-2}\)) in the depletion region on the p-side is ________.

**S1** Correct answer is 4.836.
Q47 Consider an n-channel metal oxide semiconductor field effect transistor (MOSFET) with a gate-to-source voltage of 1.8 V. Assume that \( W = 4 \), \( \mu n C_{ox} = 70 \times 10^{-6} \text{AV}^{-2} \), the threshold voltage is 0.3 V, and the channel length modulation parameter is 0.09 V\(^{-1}\). In the saturation region, the drain conductance (in micro siemens) is \_____.

S1 Correct option is 28.35.

Drain conductance in saturation region is,

\[ g_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_g - V_T) \]

\[ = \frac{1}{2} \times 70 \times 10^{-6} \times 4 \times [1.8 \text{ V} - 0.3 \text{ V}]^2 \]

\[ = 0.315 \text{ mA} \]

\[ g_D = 0.09 \times 0.315 \text{ mA/V} \]

\[ g_D = 28.35 \times 10^{-6} \text{ A/V} \]

\[ = 28.35 \mu \text{ Siemens} \]

Q48 The figure below shows the doping distribution in a P-type semiconductor in log scale.

The magnitude of the electric field (in kV/cm) in the semiconductor due to non uniform doping is \_____.

S1 Correct answer is 0.013.

\[ N_D(\text{cm}^{-3}) \]

\[ \text{Position(\mu m)} \]

The magnitude of the electric field (in kV/cm) in the semiconductor due to non uniform doping is 0.013.

Q49 Consider a silicon sample at \( T = 300 \text{ K} \), with a uniform donor density \( N_d = 5 \times 10^{16} \text{ cm}^{-3} \) illuminated uniformly such that the optical generation rate is \( G_{opt} = 1.5 \times 10^{20} \text{ cm}^{-3} \text{s}^{-1} \) through out the sample. The incident radiation is turned off at \( t = 0 \). Assume low level injection to be valid and ignore surface effects. The carrier lifetimes are \( \tau_{p0} = 0.1 \text{ and } \tau_{n0} = 0.5 \mu \text{s} \).

\[ \begin{align*}
\log_{10} x_1 & = 1 \mu \text{m} \\
\log_{10} x_2 & = 2 \mu \text{m} \\
\log_{10} x_3 & = 1 \mu \text{m} \\
\log_{10} x_4 & = 0.01 \mu \text{m} \\
\ln(10^{14}) & = 2.30 \\
\ln(10^{23}) & = 36.84 \\
\epsilon & = 0.026 \left[ 36.84 - 32.23 \right] \\
& = 0.0133 / \text{cm} \\
\end{align*} \]

The hole concentration at \( t = 0 \) and the hole concentration at \( t = 0.3 \mu \text{s} \), respectively, are

(A) \( 1.5 \times 10^{13} \text{ cm}^{-3} \) and \( 7.47 \times 10^{11} \text{ cm}^{-3} \)

(B) \( 1.5 \times 10^{13} \text{ cm}^{-3} \) and \( 8.23 \times 10^{11} \text{ cm}^{-3} \)

(C) \( 7.5 \times 10^{13} \text{ cm}^{-3} \) and \( 3.73 \times 10^{11} \text{ cm}^{-3} \)

(D) \( 7.5 \times 10^{13} \text{ cm}^{-3} \) and \( 4.12 \times 10^{11} \text{ cm}^{-3} \)

S1 Correct option is (C).

\[ P_n(t) = P_n = P_n(0) e^{-t/\tau_n} \]

At low level injection \( P_n \) negative

\[ GR = \frac{P_n(0)}{\tau_n} \]

\[ P_n(0) = GR \times \tau_n \]

\[ = 1.5 \times 10^{20} \times 0.5 \times 10^{-6} \]

\[ = 7.5 \times 10^{13} / \text{cm}^3 \]

At \( t = 0 \)

\[ P(t) = P_n(0) e^0 \]

\[ = 7.5 \times 10^{13} / \text{cm}^3 \]

At \( t = 0.3 \mu \text{s} \)

\[ P(t) = P_n(0) e^{0.3} \]

\[ = 3.73 \times 10^{13} / \text{cm}^3 \]

Q50 An idel opamp has voltage sources \( V_1, V_2, V_3, V_5, \)
... $V_{N-1}$ connected to the non-inverting input and $V_i$, $V_0$, $V_N$ connected to the inverting input as shown in the figure below ($+V_{CC} = 15$ volt, $-V_{CC} = -15$ volt). The voltage $V_1$, $V_2$, $V_3$, $V_4$, $V_5$, $V_6$, ..., are $1, -1/2, 1/3, -1/4, 1/5, -1/6, ...$ volt, respectively. As $N$ approaches infinity, the output voltage (in volts) is 

\[ V_{0} = +15 \text{ V} \]

**Q51** A p-i-n photo diode of responsivity 0.8 A/W is connected to the inverting input of an ideal opamp as shown in the figure, $+V_{CC} = 15$ V, $-V_{CC} = -15$ V. Load resistor $R_L = 10$ kΩ. If 10 μW of power is incident on the photodiode, then the value of the photocurrent (in μA) through the load is 

\[ I_L = \frac{10 \mu W}{10 \text{ kΩ}} = 8 \times 10^{-4} \text{ A} \]

\[ I_L = 800 \times 10^{-6} \text{ A} \]

\[ I_L = 800 \mu A \]

Therefore the value of photocurrent throughout the load is 

\[ -800 \mu A \]

**Q52** Identify the circuit below.

(A) Binary to Gray code converter
(B) Binary to XS3 converter
(C) Gray to Binary converter
(D) XS3 Binary converter

**S1** Correct option is (A).

(No Answer) if considering $OP_i \rightarrow IP_i$, $OP_i \rightarrow IP_i$

**S1** Correct answer is $V_0 = 15$. 

Using superposition it can be shown that the output

\[ V_0 = \left[ 1 + \frac{R_f}{R_{N}} \right] \left[ \frac{R_p}{R_{N}} V_{P3} + \frac{R_p}{R_{PN}} V_{P4} + \ldots \frac{R_p}{R_{PN}} V_{Pn} \right] 
   - \left[ \frac{R_f}{R_{N}} V_{N1} + \frac{R_f}{R_{N}} V_{N2} + \ldots \frac{R_f}{R_{N}} V_{Nn} \right] 
\]

Where $R_{N} = R_{N1} \parallel R_{N2} \parallel \ldots \parallel R_{Nn}$ and $R_{p} = R_{P1} \parallel R_{P2} \parallel R_{P3} \parallel R_{P4} \parallel R_{PN} \parallel R_{P0}$

In the problem given

\[ R_{P1} = R_{N1} = R_{N2} = \ldots = R_{Nn} = 10 \text{ kΩ} \]

\[ R_{P2} = R_{P3} = \ldots = R_{PN} = R_{P0} = 1 \text{ kΩ} \]

\[ V_0 = \left[ 1 + \frac{10k}{n} \right] \left[ \frac{1}{1+n} \right] V_{P3} + \left[ \frac{1}{1+n} \right] V_{P4} + \ldots \]

\[ - \left[ \frac{10k}{10k} V_{N1} + \frac{10k}{10k} V_{N2} + \ldots \right] \]

\[ V_0 = (V_{P3} + V_{P4} + \ldots + V_{Pn}) - (V_{N1} + V_{N2} + \ldots + V_{Nn}) \]

If the series approaches $\infty$ then

\[ V_0 = \left( 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \ldots \right) - \left( \frac{1}{2} - \frac{1}{4} - \frac{1}{6} - \ldots \right) \]

\[ = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots \]

\[ = \infty \]

This series is called harmonic series which is a divergent infinite series.

\[ V_0 = +\infty = V_{out} = +V_{CC} = +15 \text{ V} \]
### Q53
The functionality implemented by the circuit below is

(A) 2-to-1 multiplexer  
(B) 4-to-1 multiplexer  
(C) 7-to-1 multiplexer  
(D) 6-to-1 multiplexer

**S1** Correct option is (B).

### Q54
In a 8085 system, a PUSH operation requires more clock cycles than a POP operation, which one of the following options is the correct reason for this? 
(A) For POP, the data transceivers remain in the same direction as for instruction fetch (memory to processor), whereas for PUSH their direction has to be reversed  
(B) Memory write operations are slower than memory read operations in an 8085 based system.  
(C) The stack pointer needs to be pre-determined before writing registers in a PUSH, whereas a POP operation uses the address already in the stack pointer.  
(D) Order of registers has to be interchanged for a PUSH operation, whereas POP uses their natural order.

**S1** Correct option is (C).

Push takes 12T states due to pre decrement and pop takes 10T states.

### Q55
The open-loop transfer function of a unity-feedback control system is

\[
G(s) = \frac{K}{s^2 + 5s + 5}
\]

The value of \( K \) at the breakaway point of the feedback control system’s root-locus plot is _____

**S1** Correct answer is 1.25.

Breakaway point \( \frac{ds}{ds} = 0 \)

\[
\frac{d}{ds} \left( \frac{1}{s^2 - 5s + 5} \right) = 0
\]

\( s = -2.5 \) is a breakaway point

\[ K \] Value is obtain From Magnitude Condition

\[
\left| \frac{K}{s^2 + 5s + 5} \right|_{s=2.5} = 1
\]

\[
\left| \frac{K}{6.25 - 12.5 + 5} \right| = 1
\]

\[ K = 1.25 \]

### Q56
The open-loop transfer function of a unity-feedback control system is given by

\[
G(S) = \frac{K}{s(s+2)}
\]

For the peak overshoot of the closed-loop system to a unit step input to be 10%, the value of \( K \) is _______.

**S1** Correct answer is 2.87.

Given \( \%M_p = 10\% \); \( M_p = 10\% \)

\[
M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}
\]

\( 0.1 = e^{-\pi\xi/\sqrt{1-\xi^2}} \)

\[ \ln(0.1) = -\frac{\pi\xi}{\sqrt{1-\xi^2}} \]

\[ 2.3 = \frac{\pi\xi}{\sqrt{1-\xi^2}} \]

\[ \xi = 0.59 \]

Given \( G(S) = \frac{K}{s(s+2)} \)

\[ 1 + G(S) = 0 \]

\[ s^2 + 2s + K = 0 \]
\[2\varepsilon_0 \omega_n = 2\]
\[2 \times 0.59 \times \omega_n = 2\]
\[\omega_n = 1.69 \text{ rad/sec}\]
\[K = \omega_n^2 = 2.87\]

**Q57** The transfer function of a linear time invariant system is given by \(H(s) = 2s^3 - 5s^3 + 5s - 2\). The number of zeros in the right half of the s-plane is _____.

**S1** Correct answer is 3.

**RH- Criteria**
\[
\begin{align*}
&(1) +s^3 & 2 & 0 & -2 \\
&(2) -s^3 & -5 & +5 \\
&(3) +s^1 & 0(2) \\
&(3) -s^0 & -2 \\
\end{align*}
\]
3 Sign Changes
3 Roots (Zeros) in the RH-S-Plane.

**Q58** Consider a discrete memory less source with alphabet \(S = \{s_0, s_1, s_2, s_3, s_4, \ldots\}\) and respective probabilities of occurrence \(P = \{\frac{1}{3}, \frac{1}{3}, \frac{1}{6}, \frac{1}{6}, \ldots\}\). The entropy of the source (in bits) is _____.

**S1** Correct answer is 2.

\[
H = \frac{1}{2} \log_2 3 + \frac{1}{2} \log_2 4 + \frac{1}{8} \log_2 8 + \frac{1}{16} \log_2 16 + \ldots
\]
\[
H = \frac{1}{2} + \frac{1}{4} \times 2 + \frac{1}{8} \times 3 + \frac{1}{16} \times 4
\]
\[
= \sum_{n=0}^{\infty} n \left(\frac{1}{2}\right)^n = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 2
\]

**Q59** A digital communication system uses a repetition code for channel encoding/decoding. During transmission, each bit is repeated three times (0 is transmitted as 000, and 1 is transmitted as 111). It is assumed that the source puts out symbols independently and with equal probability. The decoder operates as follows: In a block of three received bits, if the number of zeros exceeds the number of ones, the decoder decides in favor of a 0, and if the number of ones exceeds the number of zeros, the decoder decides in favor of a 1. Assuming a binary symmetric channel with crossover probability \(p = 0.1\), the average probability of error is _____.

**S1** Correct answer is 0.028.

\[
P_e = P^0 + 3P^2 (1 - P)
\]
\[
P = 0.1
\]
\[
P_e = (0.1)^3 + 3 \times (0.1)^2 (1 - 0.1)
\]
\[
= 0.001 + 3 \times 0.01 \times 0.9
\]
\[
= 0.001 + 0.027
\]
\[
= 0.028
\]

**Q60** An analog pulse \(s(t)\) is transmitted over an additive white Gaussian noise (AWGN) channel. The received signal is \(r(t) = s(t) + n(t)\), where \(n(t)\) is additive white Gaussian noise with power spectral density \(N_0/2\). The received signal is passed through a filter with impulse response \(h(t)\). Let \(E_s\) and \(E_h\) denote the energies of the pulse \(s(t)\) and the filter \(h(t)\), respectively. When the signal-to-noise ratio (SNR) is maximized at the output of the filter (SNR\(_{\text{max}}\)), which of the following holds?

(A) \(E_s = E_h\); \(\text{SNR}_{\text{max}} = \frac{2E_s}{N_0}\)

(B) \(E_s = E_h\); \(\text{SNR}_{\text{max}} = \frac{E_s}{N_0}\)

(C) \(E_s > E_h\); \(\text{SNR}_{\text{max}} > \frac{2E_s}{N_0}\)

(D) \(E_s < E_h\); \(\text{SNR}_{\text{max}} = \frac{2E_s}{N_0}\)

**S1** Correct option is (A).

The impulse response of the filter is same on the signal so
\[
\text{SNR} = \frac{E_s}{N_0}
\]
\[
E_s = E_h
\]
\[
\text{SNR} = \frac{2E_s}{N_0}
\]

**Q61** The current density in a medium is given by
\[
\mathbf{J} = \frac{400 \sin \theta}{2 \pi (r^2 + 4)} \hat{r}, \text{Am}^{-2}
\]
The total current and the average current density flowing through the portion of a spherical surface \(r = 0.8 \text{ m}, \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}, 0 \leq \phi \leq 2\pi\) are given respectively by

(A) 15.09 A, 12.86 Am\(^{-2}\)

(B) 18.73 A, 13.65 Am\(^{-2}\)

(C) 12.86 A, 9.23 Am\(^{-2}\)

(D) 10.28 A, 7.56 Am\(^{-2}\)

**S1** Correct option is not given

Current density,
\[
\mathbf{J} = \frac{400 \sin \theta}{2 \pi (r^2 + 4)} \hat{r}, \text{Am}^{-2}
\]
current passing through the portion of sphere of radius \(r = 0.8 \text{ m}\) is given by
\[
I = \int \int d\mathbf{s} \quad (r = \text{constant})
\]
\[
d\mathbf{s} = r^2 \sin \theta d\theta d\phi d\mathbf{r}
\]
\[
I = \int_{\phi}^{2\pi} \int_{\theta}^{\pi} \frac{400 \sin \theta}{2 \pi (r^2 + 4)} r^2 \sin \theta d\theta d\phi
\]
\[
= \frac{400(0.8)^2}{2 \pi (0.8^2 + 4)} \left[ \left( \frac{\pi}{4} - \frac{\pi}{12} \right) - \left( \sin \left( \frac{\pi}{2} \right) - \sin \left( \frac{\pi}{6} \right) \right) \right]
\]
\[
\times (2\pi)
\]
\[
\therefore \quad I = 7.45 \text{ Amp}
\]
The average current density through the given sphere surface is
\[
J = \frac{I}{\text{Area of } r = 0.8 \text{ m sphere}}
\]
\[
= \frac{7.45}{(0.8)^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin \theta d\theta d\phi}
\]
Q62 An antenna pointing in a certain direction has a noise temperature of 50 K. The ambient temperature is 290 K. The antenna is connected to a pre-amplifier that has a noise figure of 2 dB and an available gain of 40 dB over an effective bandwidth of 12 MHz. The effective input noise temperature for the amplifier and the noise power at the output of the preamplifier, respectively, are
(A) $T = 169.36$ K and $P_{in} = 3.73 \times 10^{-10}$ W
(B) $T = 170.8$ K and $P_{in} = 4.56 \times 10^{-10}$ W
(C) $T = 182.5$ K and $P_{in} = 3.85 \times 10^{-10}$ W
(D) $T = 160.62$ K and $P_{in} = 4.6 \times 10^{-10}$ W

S1 Correct option is (A).

Q63 Two lossless X-band horn antennas are separated by a distance of 200λ. The amplitude reflection coefficients at the terminals of the transmitting and receiving antennas are 0.15 and 0.18, respectively. The maximum directivities of the transmitting and receiving antennas (over the isotropic antenna) are 18 dB and 22 dB, respectively. Assuming that the input power in the lossless transmission line connected to the antenna is 2 W, and that the antennas are perfectly aligned and polarization matched, the power (in mW) delivered to the load at the receiver is ________

S1 Correct answer is 2.99.

Q64 The electric field of a uniform plane wave travelling along the negative z direction is given by the following equation:

$$E_w = (\hat{a}_x + j\hat{a}_y)E_0 e^{jkz}$$

This wave is incident upon a receiving antenna placed at the origin and whose radiated electric field towards the incident wave is given by the following equation:

$$\vec{E}_r = (\hat{a}_x + 2\hat{a}_y)E_0 \frac{1}{r} e^{-jkr}$$

The polarization of the incident wave, the polarization of the antenna and losses due to the polarization mismatch are, respectively,
Given Electric field of incident wave is
\[ E_{\text{inc}} = (\hat{a}_x + j\hat{a}_y) E_0 e^{j\omega t} \]

at \( z = 0; \)
\[ E_{\text{inc}} = E_0 \hat{a}_x \]

At \( \omega t = \frac{\pi}{2} \)
\[ E_{\text{inc}} = E_0 (\hat{a}_y) \]

As a tip of electric field intensity is tracing a circle when time varies, hence the wave is said to be circularly polarized in clockwise direction (or) RHCP. Polarizing vector of incident wave is given by.
\[ \hat{P}_i = \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \]

radiated electric field from the antenna is
\[ E_{\text{r}} = (\hat{a}_x + 2\hat{a}_y) E_0 \frac{1}{\gamma} e^{-j\gamma \gamma} \]

at \( r = 0 \)
\[ E_{\text{r}} = E_i cos \omega t \hat{a}_x + 2E_i cos \omega t \hat{a}_y \]

(in time varying form)

As both \( x \) and \( y \) components are in-phase, hence the wave is said to be linear polarized. Polarizing vector of radiated field is \( \hat{P}_r = (\hat{a}_x + 2\hat{a}_y) \) polarizing mismatch; The polarizing mismatch is said to have, if the polarization of receiving antenna is not same on the polarization of the incident wave. The polarization loss factor (PLF) characterizes the loss of EM power to polarization mismatch.
\[ \text{PLF} = |\hat{P}_r \cdot \hat{P}_i| \]

in dB;
\[ \text{PLF (dB)} = 10 \log (\text{PLF}) \]
\[ \text{PLF} = \left| \frac{(\hat{a}_x + j\hat{a}_y)}{2} \left( \frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}} \right) \right|^2 \]
\[ = \frac{1 + \frac{2}{\sqrt{5}}}{2} = \frac{1}{2} \] (or) 0.5
\[ \text{PLF (dB)} = 10 \log 0.5 \]
\[ = -3.0102 \]

**Q65** The far-zone power density radiated by a helical antenna is approximated as:
\[ \bar{W}_{\text{rad}} = \bar{W}_{\text{average}} \approx \hat{a}_z C_0 \frac{1}{r^2} \cos^4 \theta \]

The radiated power density is symmetrical with respect to \( \phi \) and exists only in the upper hemisphere: \( 0 \leq \theta \leq \frac{\pi}{2} \); \( 0 \leq \phi \leq 2\pi \); \( C_0 \) is a constant. The power radiated by the antenna (in watts) and the maximum directivity of the antenna, respectively, are
(A) 1.5\( C_0 \), 10 dB  
(B) 1.256\( C_0 \), 10 dB  
(C) 1.256\( C_0 \), 12 dB  
(D) 1.5\( C_0 \), 12 dB

**S1** Correct option is (B).